Λ-renormalized Einstein-Schrödinger theory: an alternative to Einstein-Maxwell theory

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Λ-renormalized Einstein-Schrödinger (LRES) theory

- Vacuum general relativity can be derived from a Palatini Lagrangian density,

$$\mathcal{L}(\Gamma^\lambda_{\rho\tau}, g_{\rho\tau}) = -\frac{1}{16\pi}\left[\sqrt{-g}\, g^{\mu\nu}R_{\nu\mu}(\Gamma) + 2\Lambda_b\sqrt{-g}\,\right]. \qquad (1)$$

- Einstein-Schrödinger theory uses $\widehat{\Gamma}^\alpha_{\mu\nu}$ and $N_{\mu\nu}$ with no symmetry properties,

$$\mathcal{L}(\widehat{\Gamma}^\lambda_{\rho\tau}, N_{\rho\tau}) = -\frac{1}{16\pi}\left[\sqrt{-N}N^{-1\mu\nu}R_{\nu\mu}(\widehat{\Gamma}\,) + 2\Lambda_b\sqrt{-N}\,\right], \qquad N = \det(N_{\mu\nu}) \qquad (2)$$

- LRES theory includes Λ_z from zero-point fluctuations and allows other fields,

$$\begin{aligned}\mathcal{L}(\widehat{\Gamma}^\lambda_{\rho\tau}, N_{\rho\tau}) = & -\frac{1}{16\pi}\left[\sqrt{-N}N^{-1\mu\nu}R_{\nu\mu}(\widehat{\Gamma}\,) + 2\Lambda_b\sqrt{-N}\,\right]\\ & -\frac{1}{16\pi}2\Lambda_z\sqrt{-g} + \mathcal{L}_m(\psi_e, A_\nu, g_{\mu\nu}, \cdots) \qquad (3)\end{aligned}$$

where the "bare" $\Lambda_b \approx -\Lambda_z$ so the "physical" $\Lambda = \Lambda_b + \Lambda_z$ matches measurement, and $\mathcal{L}_m$ lacks $F^{\mu\nu}F_{\mu\nu}$ part, and the metric $g_{\mu\nu}$ and potential A_ν are defined by

$$\sqrt{-g}g^{\nu\mu} = \sqrt{-N}N^{-1(\mu\nu)}, \qquad A_\nu = \widehat{\Gamma}^\rho_{[\rho\nu]}\sqrt{-2}\,\Lambda_b^{-1/2}/6\,, \qquad (\text{with } c = G = 1). \qquad (4)$$

- $\lim\limits_{|\Lambda_z|\to\infty}\begin{pmatrix}\text{LRES}\\ \text{theory}\end{pmatrix} = \begin{pmatrix}\text{Einstein}-\text{Maxwell}\\ \text{theory}\end{pmatrix}$ but $\omega_c \sim \dfrac{1}{l_P} \;\Rightarrow\; |\Lambda_z| \sim \omega_c^4 l_P^2 \sim \dfrac{1}{l_P^2}$.

LRES theory matches measurement as well as Einstein-Maxwell theory

- Reduces to ordinary GR without electromagnetism for symmetric fields.

- Lorentz force equation is identical to that of Einstein-Maxwell theory.

- Extra terms in Einstein and Maxwell equations are $<10^{-16}$ of usual terms for worst-case $|F_{\mu\nu}|$, $|F_{\mu\nu;\alpha}|$ and $|F_{\mu\nu;\alpha;\beta}|$ accessible to measurement.

- Exact solutions:

- EM plane-wave solution is identical to that of Einstein-Maxwell theory.
- Charged solution and Reissner-Nordström sol. have tiny fractional difference: 10^{-76}@$r\!=\!Q\!=\!M\!=\!M_\odot$; 10^{-64}@$r\!=\!10^{-17}cm, Q\!=\!e, M\!=\!M_e$.

Standard tests	fractional difference from Einstein-Maxwell result
periastron advance	10^{-78}@$Q\!=\!M\!=\!M_\odot, r\!=\!4M$; 10^{-91}@$Q_1\!=\!-Q_2\!=\!e, r\!=\!a_0$
deflection of light	10^{-79}@$Q\!=\!M\!=\!M_\odot, b\!=\!4M$; 10^{-57}@$Q\!=\!e, M\!=\!M_P, b\!=\!a_0$
time delay of light	10^{-78}@$Q\!=\!M\!=\!M_\odot, b\!=\!4M$; 10^{-56}@$Q\!=\!e, M\!=\!M_P, b\!=\!a_0$

- Other Standard Model fields can be added just like Einstein-Maxwell theory:

- Energy levels of Hydrogen atom have fractional difference of $<10^{-50}$.

LRES theory avoids the problems of Einstein-Schrödinger theory

• Matches measurement as well as Einstein-Maxwell theory.

• Definitely predicts a Lorentz force:
- Usual Lorentz force equation results from divergence of Einstein equations,
- Lorentz force also results from the EIH method, with $\mathcal{L}_m = 0$.

• Avoids ghosts:
- With a cutoff frequency $\omega_c \sim 1/l_P$ we have $\Lambda_z \sim -\omega_c^4 l_P^2$ (with c=G=1),
- Ghosts are cut off because they would have $\omega_{ghost} = \sqrt{-2\Lambda_z} \sim \sqrt{2}\,\omega_c^2 l_P > \omega_c$,
- If we fully renormalize with $\omega_c \rightarrow \infty$ then $\omega_{ghost} \rightarrow \infty$, meaning no ghost.

• Well motivated:
- It's a vacuum energy renormalization of Einstein-Schrödinger theory,
- $\Lambda_z\sqrt{-g}$ term should be expected to occur as a quantization effect,
- Zero-point fluctuations are essential to QED - they cause the Casimir effect,
- $\Lambda = \Lambda_b + \Lambda_z$ is similar to mass/charge/field-strength renormalization in QED,
- $\Lambda_z\sqrt{-g}$ modification has never been considered before.

Why pursue LRES theory if it's so close to Einstein-Maxwell theory?

• It unifies gravitation and electromagnetism in a classical sense.

• Quantization of LRES theory is untried approach to quantization of gravity:
- LRES theory gets much different than Einstein-Maxwell theory as $k \to 1/l_P$,
- This could possibly fix some infinities which spoil the quantization of GR.

• LRES theory suggests untried approaches to a complete unified field theory:
- Higher dimensions, but with LRES theory instead of vacuum GR?
- Non-abelian fields, but with LRES theory instead of Einstein-Maxwell?

• We still don't have a unified field theory, 50 years after Einstein:
- Standard Model: excludes gravity, 25 parameters, not very "beautiful",
- String theory: background dependent, spin-2 particle $\Rightarrow$ GR?, $\sim 10^{500}$ versions, problems accounting for $\Lambda > 0$ and broken symmetry.

• New ideas are needed - many approaches should be pursued, not just one.

The Lagrangian Density Again

- A_ν and $F_{\mu\nu}$ are defined by (with c=G=1)

$$A_\nu = \widehat{\Gamma}^\rho_{[\rho\nu]}\sqrt{-2}\,\Lambda_b^{-1/2}/6, \tag{5}$$

$$F_{\mu\nu} = A_{\nu,\,\mu} - A_{\mu,\nu}. \tag{6}$$

- $\widehat{\Gamma}^\alpha_{\nu\mu}$ can be decomposed into $\tilde{\Gamma}^\alpha_{\nu\mu}$ with the symmetry $\tilde{\Gamma}^\alpha_{\nu\alpha} = \tilde{\Gamma}^\alpha_{\alpha\nu}$, and A_ν,

$$\tilde{\Gamma}^\alpha_{\nu\mu} = \widehat{\Gamma}^\alpha_{\nu\mu} - 2\delta^\alpha_\nu \widehat{\Gamma}^\rho_{[\rho\mu]}/3 \tag{7}$$

$$\Rightarrow \quad \widehat{\Gamma}^\alpha_{\nu\mu} = \tilde{\Gamma}^\alpha_{\nu\mu} - 2\delta^\alpha_\nu A_\mu \sqrt{-2}\,\Lambda_b^{1/2} \tag{8}$$

- The Lagrangian density (**??**) in terms of A_μ, $\tilde{\Gamma}^\alpha_{\nu\mu}$ and $\tilde{R}_{\nu\mu} = R_{\nu\mu}(\tilde{\Gamma})$ is,

$$\begin{aligned}\mathcal{L}(\widehat{\Gamma}^\lambda_{\rho\tau}, N_{\rho\tau}) = & -\frac{1}{16\pi}\left[\sqrt{-N}N^{-1\mu\nu}(\tilde{R}_{\nu\mu} + 2A_{[\nu,\mu]}\sqrt{-2}\,\Lambda_b^{1/2}) + 2\Lambda_b\sqrt{-N}\right] \\ & -\frac{1}{16\pi}2\Lambda_z\sqrt{-g} + \mathcal{L}_m(\psi_e, A_\nu, g_{\mu\nu}, \cdots)\end{aligned} \tag{9}$$

- $\tilde{R}_{\nu\mu}$ is actually the "Hermitianized Ricci tensor" with $\tilde{R}_{\mu\nu}(\tilde{\Gamma}^T) = \tilde{R}_{\nu\mu}(\tilde{\Gamma})$,

$$\tilde{R}_{\nu\mu} = \tilde{R}^{(\text{usual})}_{\nu\mu} + \frac{1}{2}\tilde{R}^{\alpha(\text{usual})}_{\ \alpha\mu\nu} = \tilde{\Gamma}^\alpha_{\nu\mu,\alpha} - \tilde{\Gamma}^\alpha_{(\nu|\alpha,|\mu)} + \tilde{\Gamma}^\rho_{\nu\mu}\tilde{\Gamma}^\alpha_{\rho\alpha} - \tilde{\Gamma}^\rho_{\nu\alpha}\tilde{\Gamma}^\alpha_{\rho\mu}. \tag{10}$$

The Einstein Equations

- $g_{\mu\nu}$ and $f_{\mu\nu}$ are defined by (with c=G=1)

$$\sqrt{-g}\,g^{\nu\mu} = \sqrt{-N}N^{-1(\mu\nu)}, \tag{11}$$
$$\sqrt{-g}f^{\nu\mu} = \sqrt{-N}N^{-1[\mu\nu]}\Lambda_b^{1/2}/\sqrt{-2}. \tag{12}$$

Inverting these definitions gives (after some effort)

$$N_{(\nu\mu)} = g_{\nu\mu} - 2\left(f_\nu{}^\alpha f_{\alpha\mu} - \frac{1}{4}\,g_{\nu\mu}f^{\rho\alpha}f_{\alpha\rho}\right)\Lambda_b^{-1} + \mathcal{O}(\Lambda_b^{-2}), \tag{13}$$
$$N_{[\nu\mu]} = f_{\nu\mu}\sqrt{-2}\,\Lambda_b^{-1/2} + \mathcal{O}(\Lambda_b^{-3/2}). \tag{14}$$

- $f_{\mu\nu}\approx F_{\mu\nu}$ comes from $\delta\mathcal{L}/\delta(\sqrt{-N}N^{-1[\mu\nu]})=0$ and $\tilde{R}_{[\nu\mu]}=\mathcal{O}(\Lambda_b^{-1/2})$ from (**??**),

$$N_{[\nu\mu]} = 2A_{[\mu,\nu]}\sqrt{-2}\,\Lambda_b^{-1/2} - \tilde{R}_{[\nu\mu]}\Lambda_b^{-1}, \tag{15}$$
$$\Rightarrow \qquad f_{\nu\mu} = A_{\mu,\nu} - A_{\nu,\,\mu} + \mathcal{O}(\Lambda_b^{-1}). \tag{16}$$

- Einstein equations come from $\delta\mathcal{L}/\delta(\sqrt{-N}N^{-1(\mu\nu)})=0$,

$$\tilde{R}_{(\nu\mu)} - \frac{1}{2}\,g_{\nu\mu}\tilde{R}^\rho_\rho = 8\pi T_{\nu\mu} - \Lambda_b\Big(N_{(\nu\mu)} - \frac{1}{2}\,g_{\nu\mu}N^\rho_\rho\Big) + \Lambda_z\,g_{\nu\mu} \tag{17}$$
$$= 8\pi T_{\nu\mu} + 2\left(f_\nu{}^\alpha f_{\alpha\mu} - \frac{1}{4}\,g_{\nu\mu}f^{\rho\alpha}f_{\alpha\rho}\right) + \Lambda g_{\nu\mu} + \mathcal{O}(\Lambda_b^{-1}). \tag{18}$$

Maxwell's Equations

- Maxwell's equations come from $\delta\mathcal{L}/\delta A_\tau = 0$ and $F_{\mu\nu} = A_{\nu,\,\mu} - A_{\mu,\nu}$,

$$
\begin{aligned}
f^{\nu\tau}{}_{;\,\nu} &= 4\pi j^\tau, && (19)\\
F_{[\mu\nu,\alpha]} &= 0, && (20)
\end{aligned}
$$

where $f_{\mu\nu} \approx F_{\mu\nu}$ and

$$
j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta\mathcal{L}_m}{\delta A_\tau}. \tag{21}
$$

- $\mathcal{L}_m$ may contain other fields just like Einstein-Maxwell theory,

$$
\begin{aligned}
j^\tau &= Q\bar{\psi}_e \gamma^\tau \psi_e && \text{for spin}-1/2, && (22)\\
j^\tau &= \rho u^\tau && \text{for classical hydrodynamics.} && (23)
\end{aligned}
$$

The Connection Equations

- Relation between $\tilde\Gamma^\alpha_{\mu\nu}$ and $N_{\mu\nu}$ like $(\sqrt{-g}g^{\tau\rho})_{;\,\beta}\!=\!0$ comes from $\delta\mathcal{L}/\delta\tilde\Gamma^\beta_{\tau\rho}\!=\!0$,

$$
\begin{aligned}
(\sqrt{-N}N^{-1\rho\tau})_{,\,\beta} + \tilde\Gamma^\tau_{\nu\beta}\sqrt{-N}N^{-1\rho\nu} + \tilde\Gamma^\rho_{\beta\nu}\sqrt{-N}N^{-1\nu\tau} - \tilde\Gamma^\alpha_{\beta\alpha}\sqrt{-N}N^{-1\rho\tau}& \\
= \frac{8\pi}{3}\sqrt{-g}\,j^{[\rho}\delta^{\tau]}_\beta\sqrt{-2}\,\Lambda_b^{-1/2}.& \qquad (24)
\end{aligned}
$$

- Solving these equations gives

$$
\begin{aligned}
\tilde\Gamma^\alpha_{(\nu\mu)} &= \frac{1}{2}\,g^{\alpha\rho}(g_{\mu\rho,\nu}+g_{\rho\nu,\mu}-g_{\nu\mu,\rho})+\mathcal{O}(\Lambda_b^{-1}), && (25)\\
\tilde\Gamma^\alpha_{[\nu\mu]} &= \mathcal{O}(\Lambda_b^{-1/2}), && (26)\\
\tilde R_{(\nu\mu)} &= R_{\nu\mu} + (\text{terms like } f^{\alpha\tau}f_{\tau(\mu;\nu);\alpha}\Lambda_b^{-1} \text{ and } f^\rho{}_{\mu;\alpha}f^\alpha{}_{\nu;\rho}\Lambda_b^{-1}), && (27)\\
\tilde R_{[\nu\mu]} &= (\text{terms like } f_{[\mu\nu,\tau];}{}^\tau\Lambda_b^{-1/2},\ f^\tau{}_{[\mu;[\nu];\tau]}\Lambda_b^{-1/2} \text{ and } j_{[\nu,\mu]}\Lambda_b^{-1/2}). && (28)
\end{aligned}
$$

$\Rightarrow$ $\tilde R_{(\nu\mu)}\!\approx\! R_{\nu\mu}$ and $f_{\nu\mu}\!\approx\! F_{\nu\mu}$ with fractional differences $<\!10^{-16}$ for worst-case $|f_{\mu\nu}|$, $|f_{\mu\nu;\alpha}|$, $|f_{\mu\nu;\alpha;\beta}|$ accessible to measurement (e.g. $10^{20}eV, 10^{34}Hz$ γ-rays).

The Generalized Contracted Bianchi Identity

- A generalized contracted Bianchi identity results from (**??**),

$$(\sqrt{-N}N^{-1\sigma\nu}\tilde{R}_{\nu\lambda}+\sqrt{-N}N^{-1\nu\sigma}\tilde{R}_{\lambda\nu})_{,\,\sigma}-\sqrt{-N}N^{-1\sigma\nu}\tilde{R}_{\nu\sigma,\,\lambda}=0. \tag{29}$$

- It may also be written in the manifestly covariant form,

$$(\sqrt{-N}N^{-1\sigma\nu}\tilde{R}_{\nu\lambda}+\sqrt{-N}N^{-1\nu\sigma}\tilde{R}_{\lambda\nu})_{;\,\sigma}-\sqrt{-N}N^{-1\sigma\nu}\tilde{R}_{\nu\sigma;\,\lambda}=0, \tag{30}$$

- Or in a third form,

$$\tilde{G}^{\sigma}{}_{\lambda;\,\sigma}=\left(\frac{3}{2}f^{\sigma\nu}\tilde{R}_{[\sigma\nu,\lambda]}+4\pi j^{\nu}\tilde{R}_{[\nu\lambda]}\right)\sqrt{-2}\,\Lambda_b^{-1/2}, \tag{31}$$

where

$$\tilde{G}_{\nu\mu}=\tilde{R}_{(\nu\mu)}-\frac{1}{2}\,g_{\nu\mu}\tilde{R}^{\rho}_{\rho}\,. \tag{32}$$

- The usual contracted Bianchi identity $2(\sqrt{-g}g^{\sigma\nu}R_{\nu\lambda})_{,\,\sigma}-\sqrt{-g}g^{\sigma\nu}R_{\nu\sigma,\,\lambda}=0$, or $G^{\sigma}{}_{\lambda;\,\sigma}=0$ is also valid.

The Lorentz Force Equation

• Lorentz force equation comes from divergence of the Einstein equations (**??**)

$$T^{\nu}_{\mu;\,\nu} = F_{\mu\nu} j^{\nu} \tag{33}$$

where

$$j^{\tau} = \frac{-1}{\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta A_{\tau}}, \tag{34}$$

$$T_{\mu\nu} = S_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} S^{\alpha}_{\alpha}, \tag{35}$$

$$S_{\mu\nu} = \frac{2\,\delta \mathcal{L}_m}{\delta(\sqrt{-g} g^{\nu\mu})}\,. \tag{36}$$

• Here we have used equations (**??**,**??**,**??**) and the following identity which can be derived using only the definitions of $g_{\mu\nu}$ and $f_{\mu\nu}$,

$$\left(N^{(\mu}{}_{\sigma)} - \frac{1}{2}\,\delta^{\mu}_{\sigma} N^{\rho}_{\rho}\right)_{;\,\mu} = \left(\frac{3}{2} f^{\nu\rho} N_{[\nu\rho,\sigma]} + f^{\nu\rho}{}_{;\nu} N_{[\rho\sigma]}\right) \sqrt{-2}\,\Lambda_b^{-1/2}. \tag{37}$$

• Covariant derivative ";" is always done using the Christoffel connection formed from the symmetric metric $g_{\mu\nu}$.

An Exact Charged Solution

- This charged solution is very close to the Reissner-Nordström solution,

$$g_{\nu\mu} = \breve{c} \begin{pmatrix} a & & & \\ & -1/a\breve{c}^2 & & \\ & & -r^2 & \\ & & & -r^2 \sin^2\theta \end{pmatrix}, \tag{38}$$

$$f_{\nu\mu} = \frac{1}{\breve{c}} \begin{pmatrix} 0 & Q/r^2 & & \\ -Q/r^2 & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \tag{39}$$

$$A_0 = \frac{Q}{r}\left[1 + \frac{M}{\Lambda_b r^3} - \frac{4Q^2}{5\Lambda_b r^4} + \mathcal{O}(\Lambda_b^{-2})\right], \tag{40}$$

where

$$a = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}\left[1 + \frac{Q^2}{10\Lambda_b r^4} + \mathcal{O}(\Lambda_b^{-2})\right], \qquad \breve{c} = \sqrt{1 - \frac{2Q^2}{\Lambda_b r^4}}. \tag{41}$$

- Additional terms are tiny for worst-case radii accessible to measurement:
  - $Q^2/\Lambda_b r^4 \sim 10^{-76}$@$r=Q=M=M_\odot$; $\sim 10^{-64}$@$r=10^{-17}cm, Q=e, M=M_e$,
  - $M/\Lambda_b r^3 \sim 10^{-76}$@$r=Q=M=M_\odot$; $\sim 10^{-70}$@$r=10^{-17}cm, Q=e, M=M_e$.

Summary of Λ-renormalized Einstein-Schrödinger theory

- $\lim_{|\Lambda_z|\to\infty}\begin{pmatrix}\text{LRES}\\ \text{theory}\end{pmatrix}=\begin{pmatrix}\text{Einstein}-\text{Maxwell}\\ \text{theory}\end{pmatrix}$ but $\omega_c\sim\dfrac{1}{l_P} \;\Rightarrow\; |\Lambda_z|\sim\omega_c^4 l_P^2\sim\dfrac{1}{l_P^2}$.
- Matches measurement as well as Einstein-Maxwell theory.
- Reduces to ordinary GR without electromagnetism for symmetric fields.
- Other Standard Model fields can be added just like Einstein-Maxwell theory.
- Avoids the problems of the original Einstein-Schrödinger theory.
- Well motivated - it's the ES theory but with a quantization effect.
- Unifies gravitation and electromagnetism in a classical sense.
- Suggests untried approaches to a complete quantized unified field theory.
- For the details see my papers: www.arxiv.org/abs/gr-qc/0310124, www.arxiv.org/abs/gr-qc/0403052, www.arxiv.org/abs/gr-qc/0411016.