Λ-renormalized Einstein-Schrödinger theory: an alternative to Einstein-Maxwell theory

James A. Shifflett
Department of Physics
WUGRAV Gravity Group
Washington University in St. Louis
One Brookings Drive, St. Louis, Missouri 63130

shifflet@hbar.wustl.edu

Midwest Relativity Meeting, MWRM-16
17-18 November, 2006
\textbf{\(\Lambda\)-renormalized Einstein-Schrödinger (LRES) theory}

- Vacuum general relativity can be derived from a Palatini Lagrangian density,
  \[ \mathcal{L}(\Gamma_{\rho\tau}, g_{\rho\tau}) = -\frac{1}{16\pi} \left[ \sqrt{-g} g^{\mu\nu} R_{\nu\mu}(\Gamma) + 2\Lambda_b \sqrt{-g} \right]. \tag{1} \]

- Einstein-Schrödinger theory uses \(\hat{\Gamma}^{\alpha}_{\mu\nu}\) and \(N_{\mu\nu}\) with no symmetry properties,
  \[ \mathcal{L}(\hat{\Gamma}_{\rho\tau}^{\alpha}, N_{\rho\tau}) = -\frac{1}{16\pi} \left[ \sqrt{-N} N^{-1\mu\nu} R_{\nu\mu}(\hat{\Gamma}) + 2\Lambda_b \sqrt{-N} \right], \quad N = \text{det}(N_{\mu\nu}) \tag{2} \]

- LRES theory includes \(\Lambda_z\) from zero-point fluctuations and allows other fields,
  \[ \mathcal{L}(\hat{\Gamma}_{\rho\tau}^{\alpha}, N_{\rho\tau}) = -\frac{1}{16\pi} \left[ \sqrt{-N} N^{-1\mu\nu} R_{\nu\mu}(\hat{\Gamma}) + 2\Lambda_b \sqrt{-N} \right] \]
  \[ -\frac{1}{16\pi} 2\Lambda_z \sqrt{-g} + \mathcal{L}_m(\psi_e, A_{\nu}, g_{\mu\nu}, \cdots) \tag{3} \]

where the “bare” \(\Lambda_b \approx -\Lambda_z\) so the “physical” \(\Lambda = \Lambda_b + \Lambda_z\) matches measurement, and \(\mathcal{L}_m\) lacks \(F_{\mu\nu}F^{\mu\nu}\) part, and the metric \(g_{\mu\nu}\) and potential \(A_{\nu}\) are defined by

\[ \sqrt{-gg^{\nu\mu}} = \sqrt{-N} N^{-1(\mu\nu)}, \quad A_{\nu} = \hat{\Gamma}_{[\rho\nu]}^{\mu} \sqrt{-2} \Lambda_b^{-1/2}/6, \quad \text{(with } c = G = 1). \tag{4} \]

- \(\lim_{|\Lambda_z| \to \infty} (\text{LRES theory}) = (\text{Einstein–Maxwell theory})\) but \(\omega_c \sim \frac{1}{l_P} \Rightarrow |\Lambda_z| \sim \omega_c^4 l_P^2 \sim \frac{1}{l_P^2}.\)
**LRES theory matches measurement as well as Einstein-Maxwell theory**

- Reduces to ordinary GR without electromagnetism for symmetric fields.

- Lorentz force equation is identical to that of Einstein-Maxwell theory.

- Extra terms in Einstein and Maxwell equations are $<10^{-16}$ of usual terms for worst-case $|F_{\mu\nu}|$, $|F_{\mu\nu;\alpha}|$ and $|F_{\mu\nu;\alpha;\beta}|$ accessible to measurement.

- Exact solutions:
  - EM plane-wave solution is identical to that of Einstein-Maxwell theory.
  - Charged solution and Reissner-Nordström sol. have tiny fractional difference:
    \[ 10^{-76} @ r = Q = M = M_\odot; \quad 10^{-64} @ r = 10^{-17} \text{cm}, Q = e, M = M_e. \]

- **Standard tests**
<table>
<thead>
<tr>
<th>fractional difference from Einstein-Maxwell result</th>
</tr>
</thead>
<tbody>
<tr>
<td>periastron advance</td>
</tr>
<tr>
<td>deflection of light</td>
</tr>
<tr>
<td>time delay of light</td>
</tr>
</tbody>
</table>

- Other Standard Model fields can be added just like Einstein-Maxwell theory:
  - Energy levels of Hydrogen atom have fractional difference of $<10^{-50}$. 
LRES theory avoids the problems of Einstein-Schrödinger theory

- Matches measurement as well as Einstein-Maxwell theory.

- Definitely predicts a Lorentz force:
  - Usual Lorentz force equation results from divergence of Einstein equations,
  - Lorentz force also results from the EIH method, with $\mathcal{L}_m = 0$.

- Avoids ghosts:
  - With a cutoff frequency $\omega_c \sim 1/l_P$ we have $\Lambda_z \sim -\omega_c^4 l_P^2$ (with $c=G=1$),
  - Ghosts are cut off because they would have $\omega_{\text{ghost}} = \sqrt{-2\Lambda_z} \sim \sqrt{2} \omega_c^2 l_P > \omega_c$,
  - If we fully renormalize with $\omega_c \to \infty$ then $\omega_{\text{ghost}} \to \infty$, meaning no ghost.

- Well motivated:
  - It’s a vacuum energy renormalization of Einstein-Schrödinger theory,
  - $\Lambda_z \sqrt{-g}$ term should be expected to occur as a quantization effect,
  - Zero-point fluctuations are essential to QED - they cause the Casimir effect,
  - $\Lambda = \Lambda_b + \Lambda_z$ is similar to mass/charge/field-strength renormalization in QED,
  - $\Lambda_z \sqrt{-g}$ modification has never been considered before.
Why pursue LRES theory if it's so close to Einstein-Maxwell theory?

- It unifies gravitation and electromagnetism in a classical sense.

- Quantization of LRES theory is untried approach to quantization of gravity:
  - LRES theory gets much different than Einstein-Maxwell theory as \( k \to 1/l_P \),
  - This could possibly fix some infinities which spoil the quantization of GR.

- LRES theory suggests untried approaches to a complete unified field theory:
  - Higher dimensions, but with LRES theory instead of vacuum GR?
  - Non-abelian fields, but with LRES theory instead of Einstein-Maxwell?

- We still don't have a unified field theory, 50 years after Einstein:
  - Standard Model: excludes gravity, 25 parameters, not very “beautiful”;
  - String theory: background dependent, spin-2 particle ⇒ GR?,
    \( \sim 10^{500} \) versions, problems accounting for \( \Lambda > 0 \) and broken symmetry.

- New ideas are needed - many approaches should be pursued, not just one.
The Lagrangian Density Again

- $A_\nu$ and $F_{\mu\nu}$ are defined by (with $c=G=1$)
  \[ A_\nu = \sqrt{-2} \Lambda_b^{-1/2} / 6, \]
  \[ F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}. \]

- $\tilde{\Gamma}_{\nu\mu}^\alpha$ can be decomposed into $\tilde{\Gamma}_{\nu\mu}^\alpha$ with the symmetry $\tilde{\Gamma}_{\nu\alpha}^\alpha = \tilde{\Gamma}_{\alpha\nu}^\alpha$, and $A_\nu$,
  \[ \tilde{\Gamma}_{\nu\mu}^\alpha = \tilde{\Gamma}_{\nu\mu}^\alpha - 2\delta^\alpha_\nu \tilde{\Gamma}^\rho_{[\rho\mu]} / 3 \]
  \[ \Rightarrow \tilde{\Gamma}_{\nu\mu}^\alpha = \tilde{\Gamma}_{\nu\mu}^\alpha - 2\delta^\alpha_\nu A_{\mu} \sqrt{-2} \Lambda_b^{1/2} \]

- The Lagrangian density (??? in terms of $A_\mu$, $\tilde{\Gamma}_{\nu\mu}^\alpha$ and $\tilde{R}_{\nu\mu} = R_{\nu\mu}(\tilde{\Gamma})$ is,
  \[ \mathcal{L}(\hat{\Gamma}_{\rho\tau}^\lambda, N_{\rho\tau}) = \frac{-1}{16\pi} \left[ \sqrt{-N} N^{-1\mu\nu} (\tilde{R}_{\nu\mu} + 2A_{[\nu,\mu]} \sqrt{-2} \Lambda_b^{1/2}) + 2\Lambda_b \sqrt{-N} \right] \]
  \[ -\frac{1}{16\pi} 2\Lambda_z \sqrt{-g} + \mathcal{L}_m(\psi, A_\nu, g_{\mu\nu}, \cdots) \]

- $\tilde{R}_{\nu\mu}$ is actually the “Hermitianized Ricci tensor” with $\tilde{R}_{\mu\nu}(\tilde{\Gamma}^T) = \tilde{R}_{\nu\mu}(\tilde{\Gamma})$,
  \[ \tilde{R}_{\nu\mu} = \tilde{R}_{\nu\mu}^{(\text{usual})} + \frac{1}{2} \tilde{R}_{\alpha\mu\nu}^{(\text{usual})} = \tilde{\Gamma}_{\nu\mu,\alpha} - \tilde{\Gamma}_{(\nu|\alpha|,\mu)} + \tilde{\Gamma}_{\nu\mu} \tilde{\Gamma}_{\rho\alpha} - \tilde{\Gamma}_{\nu\alpha} \tilde{\Gamma}_{\rho\mu}. \]
The Einstein Equations

- $g_{\mu\nu}$ and $f_{\mu\nu}$ are defined by (with $c=G=1$)
  \[
  \sqrt{-g} g^{\nu\mu} = \sqrt{-N} N^{-1(\mu\nu)}, \\
  \sqrt{-g} f^{\nu\mu} = \sqrt{-N} N^{-1[\mu\nu]} \Lambda_b^{1/2} / \sqrt{-2}.
  \]
  (11)  
  (12)

  Inverting these definitions gives (after some effort)
  \[
  N_{(\nu\mu)} = g_{\nu\mu} - 2 \left( f_\nu^\alpha f_{\alpha\mu} - \frac{1}{4} g_{\nu\mu} f^{\rho\alpha} f_{\alpha\rho} \right) \Lambda_b^{-1} + O(\Lambda_b^{-2}),
  \]
  (13)
  \[
  N_{[\nu\mu]} = f_{\nu\mu} \sqrt{-2} \Lambda_b^{-1/2} + O(\Lambda_b^{-3/2}).
  \]
  (14)

- $f_{\mu\nu} \approx F_{\mu\nu}$ comes from $\delta \mathcal{L}/\delta (\sqrt{-N} N^{-1[\mu\nu]}) = 0$ and $	ilde{R}_{[\nu\mu]} = O(\Lambda_b^{-1/2})$ from (??),
  \[
  N_{[\nu\mu]} = 2A_{[\mu,\nu]} \sqrt{-2} \Lambda_b^{-1/2} - \tilde{R}_{[\nu\mu]} \Lambda_b^{-1},
  \]
  \[
  \Rightarrow f_{\nu\mu} = A_{\mu,\nu} - A_{\nu,\mu} + O(\Lambda_b^{-1}).
  \]
  (15)  
  (16)

- Einstein equations come from $\delta \mathcal{L}/\delta (\sqrt{-N} N^{-1(\mu\nu)}) = 0$,
  \[
  \tilde{R}_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} \tilde{R}_\rho^\rho = 8\pi T_{\nu\mu} - \Lambda_b \left( N_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} N_{\rho}^\rho \right) + \Lambda_z g_{\nu\mu}
  \]
  \[
  = 8\pi T_{\nu\mu} + 2 \left( f_\nu^\alpha f_{\alpha\mu} - \frac{1}{4} g_{\nu\mu} f^{\rho\alpha} f_{\alpha\rho} \right) + \Lambda g_{\nu\mu} + O(\Lambda_b^{-1}).
  \]
  (17)  
  (18)
Maxwell’s Equations

- Maxwell’s equations come from \( \frac{\delta \mathcal{L}}{\delta A_\tau} = 0 \) and \( F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \),

\[
\begin{align*}
\delta^{\nu\tau} \frac{\delta \mathcal{L}}{\delta A_\tau} &= 4\pi j^\tau, \\
F_{[\mu\nu,\alpha]} &= 0, \\
\end{align*}
\]

where \( f_{\mu\nu} \approx F_{\mu\nu} \) and

\[
\begin{align*}
j^\tau &= \frac{-1}{\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta A_\tau}. \\
\end{align*}
\]

- \( \mathcal{L}_m \) may contain other fields just like Einstein-Maxwell theory,

\[
\begin{align*}
j^\tau &= Q\bar{\psi}\gamma^\tau \psi \\
f^\tau &= \rho u^\tau \\
\end{align*}
\]

for spin\( -1/2 \),

for classical hydrodynamics.
The Connection Equations

- Relation between $\tilde{\Gamma}^{\alpha}_{\mu \nu}$ and $N_{\mu \nu}$ like $(\sqrt{-g} g^{\tau \rho})_{,\beta} = 0$ comes from $\delta \mathcal{L} / \delta \tilde{\Gamma}^{\beta}_{\tau \rho} = 0$,

$$
(\sqrt{-NN^{-1}^{\rho \tau}})_{,\beta} + \tilde{\Gamma}^{\tau}_{\nu \beta} \sqrt{-NN^{-1}^{\nu \rho}} + \tilde{\Gamma}^{\rho}_{\beta \nu} \sqrt{-NN^{-1}^{\nu \tau}} - \tilde{\Gamma}^{\alpha}_{\beta \alpha} \sqrt{-NN^{-1}^{\rho \tau}}
= \frac{8\pi}{3} \sqrt{-g} j^{[\rho} \delta^{\tau]} \sqrt{2} \Lambda^{-1/2}. \quad (24)
$$

- Solving these equations gives

$$
\tilde{\Gamma}^{\alpha}_{(\nu \mu)} = \frac{1}{2} g^{\alpha \rho} (g_{\rho \mu, \nu} + g_{\rho \nu, \mu} - g_{\nu \mu, \rho}) + \mathcal{O}(\Lambda^{-1}), \quad (25)
$$

$$
\tilde{\Gamma}^{\alpha}_{[\nu \mu]} = \mathcal{O}(\Lambda^{-1/2}), \quad (26)
$$

$$
\tilde{R}_{(\nu \mu)} = R_{\nu \mu} + \text{(terms like } f^{\alpha \tau} f_{\tau (\mu ; \nu) ; \alpha} \Lambda^{-1} \text{ and } f^{\rho}_{\mu ; \alpha} f^{\alpha \nu} \rho \Lambda^{-1} \text{),} \quad (27)
$$

$$
\tilde{R}_{[\nu \mu]} = \text{(terms like } f_{[\mu \nu , \tau]}^{\tau} \Lambda^{-1/2} , f^{\tau}_{[\mu ; [\nu ; \tau]} \Lambda^{-1/2} \text{ and } j_{[\nu , \mu]} \Lambda^{-1/2}). \quad (28)
$$

$\Rightarrow \tilde{R}_{(\nu \mu)} \approx R_{\nu \mu}$ and $f_{\nu \mu} \approx F_{\nu \mu}$ with fractional differences $< 10^{-16}$ for worst-case $|f_{\mu \nu}|, |f_{\mu \nu ; \alpha}|, |f_{\mu \nu ; \alpha ; \beta}|$ accessible to measurement (e.g. $10^{20} eV, 10^{34} Hz \gamma$-rays).
The Generalized Contracted Bianchi Identity

• A generalized contracted Bianchi identity results from (??),
\[
(\sqrt{-NN^{-1}\sigma^{\nu} \tilde{R}_{\nu,\lambda}} + \sqrt{-NN^{-1}\nu^{\sigma} \tilde{R}_{\lambda,\nu}}),_{\sigma} - \sqrt{-NN^{-1}\sigma^{\nu} \tilde{R}_{\nu,\sigma,\lambda}} = 0.
\]

(29)

• It may also be written in the manifestly covariant form,
\[
(\sqrt{-NN^{-1}\sigma^{\nu} \tilde{R}_{\nu,\lambda}} + \sqrt{-NN^{-1}\nu^{\sigma} \tilde{R}_{\lambda,\nu}}),_{\sigma} - \sqrt{-NN^{-1}\sigma^{\nu} \tilde{R}_{\nu,\sigma,\lambda}} = 0,
\]

(30)

• Or in a third form,
\[
\tilde{G}^{\sigma}_{\lambda;\sigma} = \left(\frac{3}{2} f^{\sigma\nu} \tilde{R}_{[\sigma\nu,\lambda]} + 4\pi j^{\nu} \tilde{R}_{[\nu\lambda]}\right) \sqrt{-2} \Lambda_{b}^{-1/2},
\]

(31)

where
\[
\tilde{G}_{\nu\mu} = \tilde{R}_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} \tilde{R}^{\rho}_{\rho}.
\]

(32)

• The usual contracted Bianchi identity \(2(\sqrt{-gg^{\sigma\nu} R_{\nu,\lambda}},_{\sigma} - \sqrt{-gg^{\sigma\nu} R_{\nu,\sigma,\lambda}} = 0,\)
or \(G^{\sigma}_{\lambda;\sigma} = 0\) is also valid.
The Lorentz Force Equation

- Lorentz force equation comes from divergence of the Einstein equations (??)

\[ T_{\mu;\nu} = F_{\mu\nu} j^\nu \]  \hspace{1cm} (33)

where

\[ j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta L_m}{\delta A^\tau}, \]  \hspace{1cm} (34)

\[ T_{\mu\nu} = S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^\alpha \alpha, \]  \hspace{1cm} (35)

\[ S_{\mu\nu} = \frac{2 \delta L_m}{\delta (\sqrt{-g} g^{\mu\nu})}. \]  \hspace{1cm} (36)

- Here we have used equations (??,??,??) and the following identity which can be derived using only the definitions of \( g_{\mu\nu} \) and \( f_{\mu\nu} \),

\[ \left( N^{(\mu}_{\sigma)} - \frac{1}{2} \delta^\mu_{\sigma} N^\rho_{\rho} \right)_{;\mu} = \left( \frac{3}{2} f^\nu_{\rho\nu;\sigma \rho} + f^\nu_{\rho;\nu\sigma \rho} N^{\nu\rho}_{\sigma\rho} \right) \sqrt{-2 \Lambda_b^{-1}}. \]  \hspace{1cm} (37)

- Covariant derivative “;” is always done using the Christoffel connection formed from the symmetric metric \( g_{\mu\nu} \).
An Exact Charged Solution

- This charged solution is very close to the Reissner-Nordström solution,

\[
g_{\nu\mu} = \tilde{c} \begin{pmatrix} a & -1/ac^2 & -r^2 \\ -1/ac^2 & -r^2 & -r^2\sin^2\theta \\ -r^2 & -r^2\sin^2\theta & 0 \end{pmatrix}, \tag{38}
\]

\[
f_{\nu\mu} = \frac{1}{\tilde{c}} \begin{pmatrix} 0 & Q/r^2 \\ -Q/r^2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{39}
\]

\[
A_0 = \frac{Q}{r} \left[ 1 + \frac{M}{\Lambda_b r^3} - \frac{4Q^2}{5\Lambda_b r^4} + O(\Lambda_b^{-2}) \right], \tag{40}
\]

where

\[
a = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \left[ 1 + \frac{Q^2}{10\Lambda_b r^4} + O(\Lambda_b^{-2}) \right], \quad \tilde{c} = \sqrt{1 - \frac{2Q^2}{\Lambda_b r^4}}. \tag{41}
\]

- Additional terms are tiny for worst-case radii accessible to measurement:
  - \(Q^2/\Lambda_b r^4 \sim 10^{-76}\) @ \(r = Q = M = M_\odot\); \(\sim 10^{-64}\) @ \(r = 10^{-17}\) cm, \(Q = e, M = M_e\),
  - \(M/\Lambda_b r^3 \sim 10^{-76}\) @ \(r = Q = M = M_\odot\); \(\sim 10^{-70}\) @ \(r = 10^{-17}\) cm, \(Q = e, M = M_e\).
Summary of $\Lambda$-renormalized Einstein-Schrödinger theory

- $\lim_{|\Lambda_z| \to \infty} \left( \text{LRES theory} \right) = \left( \text{Einstein–Maxwell theory} \right)$ but $\omega_c \sim \frac{1}{l_P} \Rightarrow |\Lambda_z| \sim \omega_c^4 l_P^2 \sim \frac{1}{l_P^2}$.

- Matches measurement as well as Einstein-Maxwell theory.
- Reduces to ordinary GR without electromagnetism for symmetric fields.
- Other Standard Model fields can be added just like Einstein-Maxwell theory.
- Avoids the problems of the original Einstein-Schrödinger theory.
- Well motivated - it’s the ES theory but with a quantization effect.
- Unifies gravitation and electromagnetism in a classical sense.
- Suggests untried approaches to a complete quantized unified field theory.