Standard Model Lagrangian (including neutrino mass terms)
From *An Introduction to the Standard Model of Particle Physics, 2nd Edition*,
W. N. Cottingham and D. A. Greenwood, Cambridge University Press, Cambridge, 2007,
Extracted by J.A. Shifflett, updated from Particle Data Group tables at pdg.lbl.gov, 2 Feb 2015.

$$\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) && \text{(U(1), SU(2) and SU(3) gauge terms)}\\
& +(\bar\nu_L,\bar e_L)\,\tilde\sigma^\mu iD_\mu\begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + \bar e_R\sigma^\mu iD_\mu e_R + \bar\nu_R\sigma^\mu iD_\mu\nu_R + \text{(h.c.)} && \text{(lepton dynamical term)}\\
& -\frac{\sqrt2}{v}\left[(\bar\nu_L,\bar e_L)\,\phi M^e e_R + \bar e_R\bar M^e\bar\phi\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}\right] && \text{(electron, muon, tauon mass term)}\\
& -\frac{\sqrt2}{v}\left[(-\bar e_L,\bar\nu_L)\,\phi^* M^\nu \nu_R + \bar\nu_R\bar M^\nu\phi^T\begin{pmatrix}-e_L\\ \nu_L\end{pmatrix}\right] && \text{(neutrino mass term)}\\
& +(\bar u_L,\bar d_L)\,\tilde\sigma^\mu iD_\mu\begin{pmatrix}u_L\\ d_L\end{pmatrix} + \bar u_R\sigma^\mu iD_\mu u_R + \bar d_R\sigma^\mu iD_\mu d_R + \text{(h.c.)} && \text{(quark dynamical term)}\\
& -\frac{\sqrt2}{v}\left[(\bar u_L,\bar d_L)\,\phi M^d d_R + \bar d_R\bar M^d\bar\phi\begin{pmatrix}u_L\\ d_L\end{pmatrix}\right] && \text{(down, strange, bottom mass term)}\\
& -\frac{\sqrt2}{v}\left[(-\bar d_L,\bar u_L)\,\phi^* M^u u_R + \bar u_R\bar M^u\phi^T\begin{pmatrix}-d_L\\ u_L\end{pmatrix}\right] && \text{(up, charmed, top mass term)}\\
& +\overline{(D_\mu\phi)}D^\mu\phi - m_h^2[\bar\phi\phi - v^2/2]^2/2v^2. && \text{(Higgs dynamical and mass term)} \quad (1)
\end{aligned}$$

where (h.c.) means Hermitian conjugate of preceeding terms, $\bar\psi=\text{(h.c.)}\psi=\psi^\dagger=\psi^{*T}$, and the derivative operators are

$$D_\mu\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}=\left[\partial_\mu-\frac{ig_1}{2}B_\mu+\frac{ig_2}{2}\mathbf{W}_\mu\right]\begin{pmatrix}\nu_L\\ e_L\end{pmatrix},\quad D_\mu\begin{pmatrix}u_L\\ d_L\end{pmatrix}=\left[\partial_\mu+\frac{ig_1}{6}B_\mu+\frac{ig_2}{2}\mathbf{W}_\mu+ig\mathbf{G}_\mu\right]\begin{pmatrix}u_L\\ d_L\end{pmatrix}, \tag{2}$$

$$D_\mu\nu_R=\partial_\mu\nu_R,\quad D_\mu e_R=[\partial_\mu-ig_1B_\mu]\,e_R,\quad D_\mu u_R=\left[\partial_\mu+\frac{i2g_1}{3}B_\mu+ig\mathbf{G}_\mu\right]u_R,\quad D_\mu d_R=\left[\partial_\mu-\frac{ig_1}{3}B_\mu+ig\mathbf{G}_\mu\right]d_R, \tag{3}$$

$$D_\mu\phi=\left[\partial_\mu+\frac{ig_1}{2}B_\mu+\frac{ig_2}{2}\mathbf{W}_\mu\right]\phi. \tag{4}$$

ϕ is a 2-component complex Higgs field. Since $\mathcal{L}$ is $SU(2)$ gauge invariant, a gauge can be chosen so ϕ has the form

$$\phi^T=(0,v+h)/\sqrt2\,,\qquad <\!\phi\!>_0^T=\text{(expectation value of }\phi)=(0,v)/\sqrt2\,, \tag{5}$$

where v is a real constant such that $\mathcal{L}_\phi=\overline{(\partial_\mu\phi)}\partial^\mu\phi-m_h^2[\bar\phi\phi-v^2/2]^2/2v^2$ is minimized, and h is a residual Higgs field. B_μ, $\mathbf{W}_\mu$ and $\mathbf{G}_\mu$ are the gauge boson vector potentials, and $\mathbf{W}_\mu$ and $\mathbf{G}_\mu$ are composed of 2×2 and 3×3 traceless Hermitian matrices. Their associated field tensors are

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu,\quad \mathbf{W}_{\mu\nu}=\partial_\mu\mathbf{W}_\nu-\partial_\nu\mathbf{W}_\mu+ig_2(\mathbf{W}_\mu\mathbf{W}_\nu-\mathbf{W}_\nu\mathbf{W}_\mu)/2,\quad \mathbf{G}_{\mu\nu}=\partial_\mu\mathbf{G}_\nu-\partial_\nu\mathbf{G}_\mu+ig(\mathbf{G}_\mu\mathbf{G}_\nu-\mathbf{G}_\nu\mathbf{G}_\mu). \tag{6}$$

The non-matrix $A_\mu, Z_\mu, W^\pm_\mu$ bosons are mixtures of $\mathbf{W}_\mu$ and B_μ components, according to the weak mixing angle θ_w,

$$A_\mu=W_{11\mu}sin\theta_w+B_\mu cos\theta_w,\qquad Z_\mu=W_{11\mu}cos\theta_w-B_\mu sin\theta_w,\qquad W^+_\mu=W^{-*}_\mu=W_{12\mu}/\sqrt2, \tag{7}$$

$$B_\mu=A_\mu cos\theta_w-Z_\mu sin\theta_w,\quad W_{11\mu}=-W_{22\mu}=A_\mu sin\theta_w+Z_\mu cos\theta_w,\quad W_{12\mu}=W^*_{21\mu}=\sqrt2\,W^+_\mu,\quad sin^2\theta_w=.2315(4). \tag{8}$$

The fermions include the leptons e_R,e_L,ν_R,ν_L and quarks u_R,u_L,d_R,d_L. They all have implicit 3-component generation indices, $e_i=(e,\mu,\tau)$, $\nu_i=(\nu_e,\nu_\mu,\nu_\tau)$, $u_i=(u,c,t)$, $d_i=(d,s,b)$, which contract into the fermion mass matrices $M^e_{ij}, M^\nu_{ij}, M^u_{ij}, M^d_{ij}$, and implicit 2-component indices which contract into the Pauli matrices,

$$\sigma^\mu=\left[\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix},\begin{pmatrix}0&-i\\i&0\end{pmatrix},\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right],\quad \tilde\sigma^\mu=[\sigma^0,-\sigma^1,-\sigma^2,-\sigma^3],\quad tr(\sigma^i)=0,\quad \sigma^{\mu\dagger}=\sigma^\mu,\quad tr(\sigma^\mu\sigma^\nu)=2\delta^{\mu\nu}. \tag{9}$$

The quarks also have implicit 3-component color indices which contract into $\mathbf{G}_\mu$. So $\mathcal{L}$ really has implicit sums over 3-component generation indices, 2-component Pauli indices, 3-component color indices in the quark terms, and 2-component $SU(2)$ indices in $(\bar\nu_L,\bar e_L),(\bar u_L,\bar d_L),(-\bar e_L,\bar\nu_L),(-\bar d_L,\bar u_L),\bar\phi$, $\mathbf{W}_\mu$, $\binom{\nu_L}{e_L},\binom{u_L}{d_L},\binom{-e_L}{\nu_L},\binom{-d_L}{u_L},\phi$.

The electroweak and strong coupling constants, Higgs vacuum expectation value (VEV), and Higgs mass are,

$$g_1 = e/cos\theta_w, \quad g_2 = e/sin\theta_w, \quad g > 6.5e = g(m_\tau^2), \quad v = 246GeV(PDG) \approx \sqrt{2}\cdot 180\,GeV(CG), \quad m_h = 125.02(30)GeV \quad (10)$$

where $e = \sqrt{4\pi\alpha\hbar c} = \sqrt{4\pi/137}$ in natural units. Using (4,5) and rewriting some things gives the mass of $A_\mu, Z_\mu, W_\mu^\pm$,

$$-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) = -\frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{2}\mathcal{W}^-_{\mu\nu}\mathcal{W}^{+\mu\nu} + \begin{pmatrix}\text{higher}\\ \text{order terms}\end{pmatrix}, \quad (11)$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \quad \mathcal{W}^\pm_{\mu\nu} = D_\mu W^\pm_\nu - D_\nu W^\pm_\mu, \quad D_\mu W^\pm_\nu = [\partial_\mu \pm ieA_\mu]W^\pm_\nu, \quad (12)$$

$$D_\mu <\phi>_0 = \frac{iv}{\sqrt{2}}\begin{pmatrix} g_2 W_{12\mu}/2 \\ g_1 B_\mu/2 + g_2 W_{22\mu}/2\end{pmatrix} = \frac{ig_2 v}{2}\begin{pmatrix} W_{12\mu}/\sqrt{2} \\ (B_\mu sin\theta_w/cos\theta_w + W_{22\mu})/\sqrt{2}\end{pmatrix} = \frac{ig_2 v}{2}\begin{pmatrix} W_\mu^+ \\ -Z_\mu/\sqrt{2}\,cos\theta_w\end{pmatrix}, \quad (13)$$

$$\Rightarrow \quad m_A = 0, \quad m_{W^\pm} = g_2 v/2 = 80.425(38)GeV, \quad m_Z = g_2 v/2cos\theta_w = 91.1876(21)GeV. \quad (14)$$

Ordinary 4-component Dirac fermions are composed of the left and right handed 2-component fields,

$$e = \begin{pmatrix} e_{L1} \\ e_{R1}\end{pmatrix}, \ \nu_e = \begin{pmatrix} \nu_{L1} \\ \nu_{R1}\end{pmatrix}, \ u = \begin{pmatrix} u_{L1} \\ u_{R1}\end{pmatrix}, \ d = \begin{pmatrix} d_{L1} \\ d_{R1}\end{pmatrix}, \ \text{(electron, electron neutrino, up and down quark)} \quad (15)$$

$$\mu = \begin{pmatrix} e_{L2} \\ e_{R2}\end{pmatrix}, \ \nu_\mu = \begin{pmatrix} \nu_{L2} \\ \nu_{R2}\end{pmatrix}, \ c = \begin{pmatrix} u_{L2} \\ u_{R2}\end{pmatrix}, \ s = \begin{pmatrix} d_{L2} \\ d_{R2}\end{pmatrix}, \ \text{(muon, muon neutrino, charmed and strange quark)} \quad (16)$$

$$\tau = \begin{pmatrix} e_{L3} \\ e_{R3}\end{pmatrix}, \ \nu_\tau = \begin{pmatrix} \nu_{L3} \\ \nu_{R3}\end{pmatrix}, \ t = \begin{pmatrix} u_{L3} \\ u_{R3}\end{pmatrix}, \ b = \begin{pmatrix} d_{L3} \\ d_{R3}\end{pmatrix}, \ \text{(tauon, tauon neutrino, top and bottom quark)} \quad (17)$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0\end{pmatrix} \quad \text{where} \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2Ig^{\mu\nu}. \quad \text{(Dirac gamma matrices in chiral representation)} \quad (18)$$

The corresponding antiparticles are related to the particles according to $\psi^c = -i\gamma^2\psi^*$ or $\psi^c_L = -i\sigma^2\psi^*_R$, $\psi^c_R = i\sigma^2\psi^*_L$. The fermion charges are the coefficients of A_μ when (8,10) are substituted into either the left or right handed derivative operators (2-4). The fermion masses are the singular values of the 3×3 fermion mass matrices M^ν, M^e, M^u, M^d,

$$M^e = \mathbf{U}_L^{e\dagger}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau\end{pmatrix}\mathbf{U}_R^e, \ M^\nu = \mathbf{U}_L^{\nu\dagger}\begin{pmatrix} m_{\nu_e} & 0 & 0 \\ 0 & m_{\nu_\mu} & 0 \\ 0 & 0 & m_{\nu_\tau}\end{pmatrix}\mathbf{U}_R^\nu, \ M^u = \mathbf{U}_L^{u\dagger}\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t\end{pmatrix}\mathbf{U}_R^u, \ M^d = \mathbf{U}_L^{d\dagger}\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b\end{pmatrix}\mathbf{U}_R^d, \quad (19)$$

$$m_e = .510998910(13)MeV, \quad m_{\nu_e} \sim .001 - 2eV, \quad m_u = 1.7 - 3.1MeV, \quad m_d = 4.1 - 5.7MeV, \quad (20)$$

$$m_\mu = 105.658367(4)MeV, \quad m_{\nu_\mu} \sim .001 - 2eV, \quad m_c = 1.18 - 1.34GeV, \quad m_s = 80 - 130MeV, \quad (21)$$

$$m_\tau = 1776.84(17)MeV, \quad m_{\nu_\tau} \sim .001 - 2eV, \quad m_t = 171.4 - 174.4GeV, \quad m_b = 4.13 - 4.37GeV, \quad (22)$$

where the $\mathbf{U}$s are 3×3 unitary matrices ($\mathbf{U}^{-1} = \mathbf{U}^\dagger$). Consequently the "true fermions" with definite masses are actually linear combinations of those in $\mathcal{L}$, or conversely the fermions in $\mathcal{L}$ are linear combinations of the true fermions,

$$e'_L = \mathbf{U}^e_L e_L, \quad e'_R = \mathbf{U}^e_R e_R, \quad \nu'_L = \mathbf{U}^\nu_L \nu_L, \quad \nu'_R = \mathbf{U}^\nu_R \nu_R, \quad u'_L = \mathbf{U}^u_L u_L, \quad u'_R = \mathbf{U}^u_R u_R, \quad d'_L = \mathbf{U}^d_L d_L, \quad d'_R = \mathbf{U}^d_R d_R, \quad (23)$$

$$e_L = \mathbf{U}^{e\dagger}_L e'_L, \quad e_R = \mathbf{U}^{e\dagger}_R e'_R, \quad \nu_L = \mathbf{U}^{\nu\dagger}_L \nu'_L, \quad \nu_R = \mathbf{U}^{\nu\dagger}_R \nu'_R, \quad u_L = \mathbf{U}^{u\dagger}_L u'_L, \quad u_R = \mathbf{U}^{u\dagger}_R u'_R, \quad d_L = \mathbf{U}^{d\dagger}_L d'_L, \quad d_R = \mathbf{U}^{d\dagger}_R d'_R. \quad (24)$$

When $\mathcal{L}$ is written in terms of the true fermions, the $\mathbf{U}$s fall out except in $\bar{u}'_L\mathbf{U}^u_L\tilde{\sigma}^\mu W^\pm_\mu \mathbf{U}^{d\dagger}_L d'_L$ and $\bar{\nu}'_L\mathbf{U}^\nu_L\tilde{\sigma}^\mu W^\pm_\mu \mathbf{U}^{e\dagger}_L e'_L$. Because of this, and some absorption of constants into the fermion fields, all the parameters in the $\mathbf{U}$s are contained in only four components of the Cabibbo-Kobayashi-Maskawa matrix $\mathbf{V}^q = \mathbf{U}^u_L\mathbf{U}^{d\dagger}_L$ and four components of the Pontecorvo-Maki-Nakagawa-Sakata matrix $\mathbf{V}^l = \mathbf{U}^\nu_L\mathbf{U}^{e\dagger}_L$. The unitary matrices $\mathbf{V}^q$ and $\mathbf{V}^l$ are often parameterized as

$$\mathbf{V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix} e^{-i\delta/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta/2}\end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13}\end{pmatrix}\begin{pmatrix} e^{i\delta/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta/2}\end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1\end{pmatrix}, \quad c_j = \sqrt{1 - s_j^2}, \quad (25)$$

$$\delta^q = 69(4)\,\text{deg}, \quad s^q_{12} = 0.2253(7), \quad s^q_{23} = 0.041(1), \quad s^q_{13} = 0.0035(2), \quad (26)$$

$$\delta^l = ?, \quad s^l_{12} = 0.560(16), \quad s^l_{23} = 0.7(1), \quad s^l_{13} = 0.153(28). \quad (27)$$

$\mathcal{L}$ is invariant under a $U(1)\otimes SU(2)$ gauge transformation with $U^{-1} = U^\dagger$, $detU = 1$, θ real,

$$\mathbf{W}_\mu \to U\mathbf{W}_\mu U^\dagger - (2i/g_2)U\partial_\mu U^\dagger, \quad \mathbf{W}_{\mu\nu} \to U\mathbf{W}_{\mu\nu}U^\dagger, \quad B_\mu \to B_\mu + (2/g_1)\partial_\mu\theta, \quad B_{\mu\nu} \to B_{\mu\nu}, \quad \phi \to e^{-i\theta}U\phi, \quad (28)$$

$$\begin{pmatrix}\nu_L \\ e_L\end{pmatrix} \to e^{i\theta}U\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}, \quad \begin{pmatrix}u_L \\ d_L\end{pmatrix} \to e^{-i\theta/3}U\begin{pmatrix}u_L \\ d_L\end{pmatrix}, \quad \begin{matrix}\nu_R \to \nu_R, & u_R \to e^{-4i\theta/3}u_R, \\ e_R \to e^{2i\theta}e_R, & d_R \to e^{2i\theta/3}d_R,\end{matrix} \quad (29)$$

and under an $SU(3)$ gauge transformation with $V^{-1} = V^\dagger$, $detV = 1$,

$$\mathbf{G}_\mu \to V\mathbf{G}_\mu V^\dagger - (i/g)V\partial_\mu V^\dagger, \quad \mathbf{G}_{\mu\nu} \to V\mathbf{G}_{\mu\nu}V^\dagger, \quad u_L \to Vu_L, \quad d_L \to Vd_L, \quad u_R \to Vu_R, \quad d_R \to Vd_R. \quad (30)$$