A modification of Einstein-Schrödinger theory which closely approximates Einstein-Weinberg-Salam theory

James A. Shifflett
Department of Physics
WUGRAV Gravity Group
Washington University in St. Louis

shifflet@hbar.wustl.edu

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Comparison of the Lagrangians

- Einstein-Weinberg-Salam theory can be derived from a Palatini Lagrangian

$$\begin{aligned}\mathcal{L}(\Gamma^\lambda_{\rho\tau}, g_{\rho\tau}, \mathcal{A}_\nu) &= -\frac{1}{16\pi}\sqrt{-g}\,[\,g^{\mu\nu}R_{\nu\mu}(\Gamma)+2\Lambda_b\,]\\ &+\frac{1}{32\pi}\sqrt{-g}\;tr(f_{\rho\alpha}g^{\alpha\mu}g^{\rho\nu}f_{\nu\mu})+\mathcal{L}_m(g_{\mu\nu},\mathcal{A}_\nu,\psi_{e\nu},\phi\cdots), \qquad (1)\end{aligned}$$

where $\mathcal{A}_\nu = Ia_\nu+\sigma_i b^i_\nu$ is composed of 2×2 Hermitian matrix components and

$$f_{\nu\mu} = 2\mathcal{A}_{[\mu,\nu]}+\frac{i\sqrt{\alpha}}{2\,l_P sin\theta_w}[\mathcal{A}_\nu,\mathcal{A}_\mu]. \qquad (2)$$

- Non-Abelian LRES theory allows nonsymmetric $N_{\mu\nu}$ and $\widehat{\Gamma}^\lambda_{\rho\tau}$ with 2×2 Hermitian matrix components, excludes $tr(f_{\rho\alpha}g^{\alpha\mu}g^{\rho\nu}f_{\nu\mu})$, and includes a Λ_z,

$$\begin{aligned}\mathcal{L}(\widehat{\Gamma}^\alpha_{\rho\tau}, N_{\rho\tau}) &= -\frac{1}{16\pi}N^{1/4}[\,tr(N^{-1\mu\nu}\mathcal{R}_{\nu\mu}(\widehat{\Gamma}))+4\Lambda_b\,]\\ &-\frac{1}{4\pi}\mathrm{g}^{1/4}\Lambda_z+\mathcal{L}_m(g_{\mu\nu},\mathcal{A}_\nu,\psi_{e\nu},\phi\cdots), \qquad N=\det(N_{\mu\nu}) \qquad (3)\end{aligned}$$

where the "bare" $\Lambda_b\approx-\Lambda_z$ so that $\Lambda=\Lambda_b+\Lambda_z$ matches measurement, and

$$\mathcal{A}_\nu=\widehat{\Gamma}^\sigma_{[\nu\sigma]}/\sqrt{-18\Lambda_b}, \quad \mathrm{g}^{1/4}\mathrm{g}^{\mu\nu}=N^{1/4}N^{-1(\mu\nu)}, \quad (\mathrm{g}_{\nu\mu}=Ig_{\nu\mu}\text{ assumed}). \qquad (4)$$

The field equations

- The electro-weak field tensor $f^{\nu\mu}$ is defined by

$$\mathrm{g}^{1/4} f^{\nu\mu} \;=\; i N^{1/4} N^{-1[\nu\mu]} \Lambda_b^{1/2}/\sqrt{2}, \qquad (5)$$

- Ampere's law is identical to Weinberg-Salam theory

$$(\mathrm{g}^{1/4} f^{\omega\tau})_{,\,\omega} - \sqrt{-2\Lambda_b}\,\mathrm{g}^{1/4}[f^{\omega\tau}, \mathcal{A}_\omega] = 4\pi \mathrm{g}^{1/4} j^\tau \qquad (6)$$

- This determines the value of Λ_b and Λ_z,

$$-\Lambda_z \approx \Lambda_b = \frac{\alpha}{8\, l_P^2 sin^2\theta_w} \sim 1/l_P^2, \qquad (7)$$

and this is consistent with Λ_z being caused by zero-point fluctuations

- Other field equations have tiny extra terms

$$f_{\nu\mu} = 2\mathcal{A}_{[\mu,\nu]} + \sqrt{-2\Lambda_b}\,[\mathcal{A}_\nu, \mathcal{A}_\mu] + \underline{(f^2)\Lambda_b^{-1/2} + (f'')\Lambda_b^{-1}\cdots} \qquad (8)$$

$$G_{\nu\mu} = 8\pi T_{\nu\mu} + tr\left(f^\sigma{}_{(\nu} f_{\mu)\sigma} - \frac{1}{4}\, g_{\nu\mu} f^{\rho\sigma} f_{\sigma\rho}\right) + \Lambda g_{\nu\mu} + \underline{(f^3)\Lambda_b^{-1/2} + (f' f')\Lambda_b^{-1}\ldots} \qquad (9)$$

Non-Abelian LRES theory $\approx$ Einstein-Weinberg-Salam theory

- Extra terms in the field equations are $< 10^{-13}$ of usual terms for worst-case $|f_{\mu\nu}|$, $|f_{\mu\nu;\alpha}|$ and $|f_{\mu\nu;\alpha;\beta}|$ accessible to measurement.

- Abelian LRES theory closely approximates Einstein-Maxwell theory: see Gen. Rel. Grav. (Online First), gr-qc/0801.2307.

- Usual Lorentz-force equation results from divergence of Einstein equations

- Exact solutions:

- EM plane-wave solution is identical to that of Einstein-Maxwell theory.
- Charged solution and Reissner-Nordström sol. have tiny fractional difference:
  10^{-76} for $r = Q = M = M_\odot$, 10^{-64} for $r = 10^{-17}cm, Q = e, M = M_e$.

- Standard tests | fractional difference from Einstein-Maxwell result

test case $\rightarrow$	$Q = M = M_\odot, r = 4M$	$Q = e, M = M_P, r = a_0$
periastron advance	10^{-78}	10^{-91}
deflection of light	10^{-79}	10^{-57}
time delay of light	10^{-78}	10^{-56}

- Possible Proca field ghost with $M_{Proca}/\hbar = \sqrt{2\Lambda_b} \sim 1/l_P$, but probably not

Lagrangian has $U(1)\otimes SU(2)$ invariance like Weinberg-Salam theory

- $\mathcal{L}\rightarrow\mathcal{L}$ under $SU(2)$ gauge transformation, with 2×2 matrix U

$$\mathcal{A}_\nu\rightarrow U\mathcal{A}_\nu U^{-1}+\frac{i}{\sqrt{2\Lambda_b}}U_{,\nu}U^{-1}, \tag{10}$$

$$\Rightarrow\quad \widehat{\Gamma}^\alpha_{\nu\mu}\rightarrow U\widehat{\Gamma}^\alpha_{\nu\mu}U^{-1}+2\delta^\alpha_{[\nu}U_{,\mu]}U^{-1}, \tag{11}$$

$$\Rightarrow\quad \widehat{\mathcal{R}}_{\nu\mu}\rightarrow U\widehat{\mathcal{R}}_{\nu\mu}U^{-1} \tag{12}$$

$$N_{\nu\mu}\rightarrow UN_{\nu\mu}U^{-1},\quad \mathsf{g}_{\nu\mu}\rightarrow U\mathsf{g}_{\nu\mu}U^{-1},\quad f_{\nu\mu}\rightarrow Uf_{\nu\mu}U^{-1}. \tag{13}$$

- $\mathcal{L}\rightarrow\mathcal{L}$ under $U(1)$ gauge transformation, with scalar φ

$$\mathcal{A}_\nu\rightarrow\mathcal{A}_\nu+\frac{1}{\sqrt{2\Lambda_b}}\varphi_{,\nu}, \tag{14}$$

$$\Rightarrow\quad \widehat{\Gamma}^\alpha_{\nu\mu}\rightarrow\widehat{\Gamma}^\alpha_{\nu\mu}-2iI\delta^\alpha_{[\nu}\varphi_{,\mu]}, \tag{15}$$

$$\Rightarrow\quad \widehat{\mathcal{R}}_{\nu\mu}\rightarrow\widehat{\mathcal{R}}_{\nu\mu} \tag{16}$$

$$N_{\nu\mu}\rightarrow N_{\nu\mu},\quad \mathsf{g}_{\nu\mu}\rightarrow\mathsf{g}_{\nu\mu},\quad f_{\nu\mu}\rightarrow f_{\nu\mu}. \tag{17}$$

- $\mathcal{L}^*=\mathcal{L}$ when $\mathcal{A}_\nu$ and $f_{\nu\mu}$ are Hermitian

$$\widehat{\Gamma}^{\alpha\,*}_{\nu\mu}=\widehat{\Gamma}^{\alpha\,T}_{\mu\nu},\quad \widehat{\mathcal{R}}^*_{\nu\mu}=\widehat{\mathcal{R}}^T_{\mu\nu},\quad N^*_{\nu\mu}=N^T_{\mu\nu},\quad N^*=N, \tag{18}$$

$$\Rightarrow\quad \mathcal{A}^*_\nu=\mathcal{A}^T_\nu,\qquad f^*_{\nu\mu}=f^T_{\nu\mu},\qquad \mathsf{g}^*_{\nu\mu}=\mathsf{g}^T_{\nu\mu},\quad \mathsf{g}^*=\mathsf{g}. \tag{19}$$

Summary of non-Abelian Λ-renormalized Einstein-Schrödinger theory

- $\binom{\text{non}-\text{Abelian}}{\text{LRES theory}} \approx \binom{\text{Einstein}-\text{Weinberg}-}{\text{Salam theory}}$ with $|\Lambda_z| \approx \frac{\alpha}{8\, l_P^2 sin^2\theta_w} \sim \frac{1}{l_P^2}$.
- Extra terms in Weinberg-Salam field equations are $< 10^{-13}$ of usual terms.
- $\lim_{|\Lambda_z|\to\infty} \binom{\text{LRES}}{\text{theory}} = \binom{\text{Einstein}-\text{Maxwell}}{\text{theory}}$ $\left(\begin{matrix}\text{Gen.Rel.Grav.(Online First)}\\ \text{gr}-\text{qc/0801.2307}\end{matrix}\right)$
- Lagrangian has $U(1)\otimes SU(2)$ invariance like Weinberg-Salam theory.
- $\mathcal{L}_m$ contains $\psi_{e\nu}, \phi$ fields, and could also include rest of Standard Model.
- It's the ES theory but with Hermitian matrix components and a Λ_z term.
- Suggests untried approaches to a complete unified field theory:

- Larger matrices: 5×5 matrices for $SU(5)$ or $U(1)\otimes SU(5)$?

- For details see gr-qc/0804.1962

Backup charts

The non-Abelian Ricci tensor

• We use one of many nonsymmetric generalizations of Ricci tensor

$$\hat{\mathcal{R}}_{\nu\mu} = \widehat{\Gamma}^{\alpha}_{\nu\mu,\alpha} - \widehat{\Gamma}^{\alpha}_{(\alpha(\nu),\mu)} + \frac{1}{2}\widehat{\Gamma}^{\sigma}_{\nu\mu}\widehat{\Gamma}^{\alpha}_{(\sigma\alpha)} + \frac{1}{2}\widehat{\Gamma}^{\alpha}_{(\sigma\alpha)}\widehat{\Gamma}^{\sigma}_{\nu\mu} - \widehat{\Gamma}^{\sigma}_{\nu\alpha}\widehat{\Gamma}^{\alpha}_{\sigma\mu} - \widehat{\Gamma}^{\tau}_{[\tau\nu]}\widehat{\Gamma}^{\rho}_{[\rho\mu]}/3 \qquad (20)$$

• Because it has important invariance properties

$$R_{\rho\tau}(\widehat{\Gamma}^{\alpha\,T}_{\nu\mu}) \;=\; R^{T}_{\tau\rho}(\widehat{\Gamma}^{\alpha}_{\mu\nu}) \qquad \text{(transposition symmetric)} \quad (21)$$

$$R_{\rho\tau}(U\widehat{\Gamma}^{\alpha}_{\nu\mu}U^{-1} + 2\delta^{\alpha}_{[\nu}U_{,\mu]}U^{-1}) \;=\; U R_{\rho\tau}(\widehat{\Gamma}^{\alpha}_{\nu\mu})U^{-1} \qquad \text{(almost SU(2) invariant)} \quad (22)$$

$$R_{\rho\tau}(\widehat{\Gamma}^{\alpha}_{\nu\mu} - 2iI\,\delta^{\alpha}_{[\nu}\varphi_{,\mu]}) \;=\; R_{\rho\tau}(\widehat{\Gamma}^{\alpha}_{\nu\mu}) \qquad \text{(U(1) invariant)} \quad (23)$$

• For Abelian fields the third and fourth terms are the same.

• Reduces to the ordinary Ricci tensor for $\widehat{\Gamma}^{\alpha}_{[\nu\mu]} = 0$ and $\widehat{\Gamma}^{\alpha}_{\alpha[\nu,\mu]} = 0$, as occurs in ordinary general relativity.

The Lagrangian Density Again

- $\mathcal{A}_\nu$ is defined by

$$\mathcal{A}_\nu = \widehat{\Gamma}^{\rho}_{[\nu\rho]}/\sqrt{-18\Lambda_b}\,. \tag{24}$$

- $\widehat{\Gamma}^{\alpha}_{\nu\mu}$ can be decomposed into $\tilde{\Gamma}^{\alpha}_{\nu\mu}$ with the symmetry $\tilde{\Gamma}^{\alpha}_{\nu\alpha}=\tilde{\Gamma}^{\alpha}_{\alpha\nu}$, and $\mathcal{A}_\nu$,

$$\tilde{\Gamma}^{\alpha}_{\nu\mu} = \widehat{\Gamma}^{\alpha}_{\nu\mu} + (\delta^{\alpha}_{\mu}\widehat{\Gamma}^{\sigma}_{[\sigma\nu]} - \delta^{\alpha}_{\nu}\widehat{\Gamma}^{\sigma}_{[\sigma\mu]})/3 \quad \Rightarrow \quad \widehat{\Gamma}^{\alpha}_{\nu\mu} = \tilde{\Gamma}^{\alpha}_{\nu\mu} + 2\delta^{\alpha}_{[\mu}\mathcal{A}_{\nu]}\sqrt{-2\Lambda_b}\,. \tag{25}$$

- The Lagrangian density (3) in terms of $\mathcal{A}_\mu$, $\tilde{\Gamma}^{\alpha}_{\nu\mu}$ and $\tilde{\mathcal{R}}_{\nu\mu}=\mathcal{R}_{\nu\mu}(\tilde{\Gamma})$ is,

$$\begin{aligned}\mathcal{L}(\widehat{\Gamma}^{\lambda}_{\rho\tau}, N_{\rho\tau}) = & -\frac{1}{16\pi}N^{1/4}\Big[tr(N^{-1\mu\nu}(\tilde{\mathcal{R}}_{\nu\mu} + 2\mathcal{A}_{[\nu,\mu]}\sqrt{-2\Lambda_b}\,) + 2\Lambda_b[\mathcal{A}_\nu,\mathcal{A}_\mu]) + 4\Lambda_b\Big] \\ & -\frac{1}{4\pi}\Lambda_z \mathrm{g}^{1/4} + \mathcal{L}_m(\mathcal{A}_\nu, g_{\mu\nu}, \psi_{e\nu}, \phi, \cdots).\end{aligned} \tag{26}$$

- The nonsymmetric Ricci tensor (20) reduces to

$$\tilde{\mathcal{R}}_{\nu\mu} = \tilde{\Gamma}^{\alpha}_{\nu\mu,\alpha} - \tilde{\Gamma}^{\alpha}_{\alpha(\nu,\mu)} + \frac{1}{2}\tilde{\Gamma}^{\sigma}_{\nu\mu}\tilde{\Gamma}^{\alpha}_{\sigma\alpha} + \frac{1}{2}\tilde{\Gamma}^{\alpha}_{\sigma\alpha}\tilde{\Gamma}^{\sigma}_{\nu\mu} - \tilde{\Gamma}^{\sigma}_{\nu\alpha}\tilde{\Gamma}^{\alpha}_{\sigma\mu} \tag{27}$$

- We assume the special case $\mathrm{g}_{\nu\mu} = Itr(\mathrm{g}_{\nu\mu})/2$ and $\tilde{\Gamma}^{\alpha}_{\nu\mu} = Itr(\tilde{\Gamma}^{\alpha}_{\nu\mu})/2$.

The Einstein Equations

- $\mathsf{g}_{\mu\nu}$ and $f_{\mu\nu}$ are defined by (with c=G=1)

$$\mathsf{g}^{1/4}\mathsf{g}^{\nu\mu} = N^{1/4}N^{-1(\mu\nu)} \tag{28}$$

$$\mathsf{g}^{1/4}f^{\nu\mu} = iN^{1/4}N^{-1[\nu\mu]}\Lambda_b^{1/2}/\sqrt{2} \tag{29}$$

Inverting these definitions gives (after some effort)

$$N_{(\nu\mu)} = \mathsf{g}_{\nu\mu} - 2\left(f^{\sigma}{}_{(\nu}f_{\mu)\sigma} - \frac{1}{4}\,\mathsf{g}_{\nu\mu}tr(f^{\rho}{}_{\sigma}f^{\sigma}{}_{\rho})/2\right)\Lambda_b^{-1} + (f^3)\Lambda_b^{-3/2}\cdots \tag{30}$$

$$N_{[\nu\mu]} = f_{\nu\mu}\sqrt{2}\,i\,\Lambda_b^{-1/2} + (f^2)\Lambda_b^{-1}\cdots \tag{31}$$

- $f_{\mu\nu}$ comes from $\delta\mathcal{L}/\delta(\sqrt{-N}N^{-1[\mu\nu]})=0$ and $\tilde{\mathcal{R}}_{[\nu\mu]}=(f'')\Lambda_b^{-1/2}\cdots$ from (43),

$$N_{[\nu\mu]} = 2\mathcal{A}_{[\mu,\nu]}\sqrt{-2}\,\Lambda_b^{-1/2} - 2[\mathcal{A}_\nu,\mathcal{A}_\mu] - \tilde{\mathcal{R}}_{[\nu\mu]}\Lambda_b^{-1} \tag{32}$$

$$\Rightarrow \qquad f_{\nu\mu} = \mathcal{A}_{\mu,\nu} - \mathcal{A}_{\nu,\mu} + \sqrt{-2\Lambda_b}\,[\mathcal{A}_\nu,\mathcal{A}_\mu] + (f^2)\Lambda_b^{-1/2} + (f'')\Lambda_b^{-1}\cdots \tag{33}$$

- Einstein equations come from $\delta\mathcal{L}/\delta(\sqrt{-N}N^{-1(\mu\nu)})=0$,

$$\tilde{G}_{\nu\mu} = 8\pi T_{\nu\mu} - \Lambda_b tr\left(N_{(\nu\mu)} - \frac{1}{2}\,\mathsf{g}_{\nu\mu}N^{\rho}_{\rho}\right) + \Lambda_z\,\mathsf{g}_{\nu\mu} \tag{34}$$

$$= 8\pi T_{\nu\mu} + tr\left(f^{\sigma}{}_{(\nu}f_{\mu)\sigma} - \frac{1}{4}\,g_{\nu\mu}f^{\rho\sigma}f_{\sigma\rho}\right) + \Lambda g_{\nu\mu} + (f^3)\Lambda_b^{-1/2} + (f'f')\Lambda_b^{-1}\cdots \tag{35}$$

Weinberg-Salam equivalent of Ampere's Law

- Maxwell's equations come from $\delta\mathcal{L}/\delta\mathcal{A}_\tau = 0$,

$$(\mathrm{g}^{1/4} f^{\omega\tau})_{,\omega} - \sqrt{-2\Lambda_b}\,\mathrm{g}^{1/4}[f^{\omega\tau}, \mathcal{A}_\omega] = 4\pi\mathrm{g}^{1/4} j^\tau, \tag{36}$$

where $f_{\nu\mu} \approx 2\mathcal{A}_{[\mu,\nu]} + \sqrt{-2\Lambda_b}\,[\mathcal{A}_\nu, \mathcal{A}_\mu]$ and

$$j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta\mathcal{L}_m}{\delta\mathcal{A}_\tau}. \tag{37}$$

- $\mathcal{L}_m$ contains $\psi_{e\nu}, \phi$ fields of Weinberg-Salam theory,

$$j^\tau = Q\bar{\psi}_{e\nu}\gamma^\tau\psi_{e\nu}. \tag{38}$$

The Connection Equations

- Relation between $\tilde{\Gamma}^\alpha_{\mu\nu}$ and $N_{\mu\nu}$ like $(\sqrt{-g}g^{\tau\rho})_{;\,\beta}\!=\!0$ comes from $\delta\mathcal{L}/\delta\tilde{\Gamma}^\beta_{\tau\rho}\!=\!0$,

$$tr[(N^{1/4}N^{-1\rho\tau})_{,\,\beta}+\tilde{\Gamma}^\tau_{\sigma\beta}N^{1/4}N^{-1\rho\sigma}+\tilde{\Gamma}^\rho_{\beta\sigma}N^{1/4}N^{-1\sigma\tau}-\tilde{\Gamma}^\alpha_{\beta\alpha}N^{1/4}N^{-1\rho\tau}]$$
$$=\frac{8\pi\sqrt{2}\,i}{3}\mathrm{g}^{1/4}\,tr[j^{[\rho}]\delta^{\tau]}_\beta\Lambda_b^{-1/2}. \qquad (39)$$

- Solving these equations gives

$$\tilde{\Gamma}^\alpha_{(\nu\mu)} = \frac{I}{2}g^{\alpha\rho}(g_{\mu\rho,\nu}+g_{\rho\nu,\mu}-g_{\nu\mu,\rho})+(f'f)\Lambda_b^{-1}\cdots, \qquad (40)$$
$$\tilde{\Gamma}^\alpha_{[\nu\mu]} = (f')\Lambda_b^{-1}\cdots, \qquad (41)$$
$$\tilde{\mathcal{R}}_{(\nu\mu)} = R_{\nu\mu}+(f'f')\Lambda_b^{-1}\cdots, \qquad (42)$$
$$\tilde{\mathcal{R}}_{[\nu\mu]} = (f'')\Lambda_b^{-1/2}\cdots. \qquad (43)$$

$\Rightarrow \tilde{\mathcal{R}}_{(\nu\mu)}\!\approx\! R_{\nu\mu}$ and $f_{\nu\mu}\!\approx\!2\mathcal{A}_{[\mu,\nu]}+\sqrt{-2\Lambda_b}\,[\mathcal{A}_\nu,\mathcal{A}_\mu]$ with fractional differences $<10^{-13}$ for worst-case $|f_{\mu\nu}|$, $|f_{\mu\nu;\alpha}|$, $|f_{\mu\nu;\alpha;\beta}|$ accessible to measurement (e.g. $10^{20}eV, 10^{34}Hz$ γ-rays).

Proca waves as Pauli-Villars ghosts?

- If wave-packet Proca waves exist and if they have negative energy, perhaps the Proca field functions as a built-in Pauli-Villars ghost

$$\omega_c = \omega_{Proca} = \sqrt{2\Lambda_b} \ , \qquad -\Lambda_z \approx \Lambda_b = \frac{\alpha}{8 l_P^2 sin^2\theta_w} \tag{44}$$

$$\Lambda_z = -\frac{\omega_c^4 l_P^2}{2\pi}\left(\begin{matrix}\text{fermion}\\ \text{spin states}\end{matrix} - \begin{matrix}\text{boson}\\ \text{spin states}\end{matrix}\right) \tag{45}$$

$$\Rightarrow \qquad \left(\begin{matrix}\text{fermion}\\ \text{spin states}\end{matrix} - \begin{matrix}\text{boson}\\ \text{spin states}\end{matrix}\right) = \frac{4\pi sin^2\theta_w}{\alpha} = 412.8 \pm 2 \tag{46}$$

- In this case $4\pi sin^2\theta_w/\alpha$ or its "bare" value at ω_c should be an integer.
- For the Standard Model the difference in (46) is about 60.
- Non-Abelian LRES theory works for $d \times d$ instead of 2×2 matrices.
- Perhaps some value of "d" is consistent with (46).
- $SU(5)$ almost unifies the Standard Model, how about $U(1) \otimes SU(5)$?