A modification of Einstein-Schrödinger theory which closely approximates Einstein-Weinberg-Salam theory

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Comparison of the Lagrangians

• Einstein-Weinberg-Salam theory can be derived from a Palatini Lagrangian
\[ \mathcal{L}(\Gamma_{\rho\tau}^\lambda, g_{\rho\tau}, A_\nu) = -\frac{1}{16\pi} \sqrt{-g} \left[ g^{\mu\nu} R_{\nu\mu}(\Gamma) + 2\Lambda_b \right] \]
\[ + \frac{1}{32\pi} \sqrt{-g} \, tr(g^{\alpha\mu} g^{\rho\nu} f_{\nu\mu}) + \mathcal{L}_m(g_{\mu\nu}, A_\nu, \psi_{e\nu}, \phi \cdots), \quad (1) \]
where \( A_\nu = I a_\nu + \sigma_i b_i^\nu \) is composed of 2x2 Hermitian matrix components and
\[ f_{\nu\mu} = 2 A_{[\mu,\nu]} + \frac{i \sqrt{\alpha}}{2 l_P sin\theta_w} [A_\nu, A_\mu]. \quad (2) \]

• Non-Abelian LRES theory allows nonsymmetric \( N_{\mu\nu} \) and \( \hat{\Gamma}_{\rho\tau}^\lambda \) with 2x2 Hermitian matrix components, excludes \( tr(g^{\alpha\mu} g^{\rho\nu} f_{\nu\mu}) \), and includes a \( \Lambda_z \),
\[ \mathcal{L}(\hat{\Gamma}_{\rho\tau}^\alpha, N_{\rho\tau}) = -\frac{1}{16\pi} N^{1/4} \left[ tr(N^{-1}_{\mu\nu} R_{\nu\mu}(\hat{\Gamma})) + 4\Lambda_b \right] \]
\[ - \frac{1}{4\pi} g^{1/4} \Lambda_z + \mathcal{L}_m(g_{\mu\nu}, A_\nu, \psi_{e\nu}, \phi \cdots), \quad N = \det(N_{\mu\nu}) \quad (3) \]
where the “bare” \( \Lambda_b \approx -\Lambda_z \) so that \( \Lambda = \Lambda_b + \Lambda_z \) matches measurement, and
\[ A_\nu = \hat{\Gamma}_{[\nu\sigma]}^\rho / \sqrt{-18\Lambda_b}, \quad g^{1/4} g^{\mu\nu} = N^{1/4} N^{-1(\mu\nu)}, \quad (g_{\nu\mu} = I g_{\nu\mu} \text{ assumed}). \quad (4) \]
The field equations

- The electro-weak field tensor $f^{\nu\mu}$ is defined by
  \[ g^{1/4} f^{\nu\mu} = i N^{1/4} N^{-1[\nu\mu]} \Lambda_b^{1/2} / \sqrt{2}, \]  
  \[ (5) \]

- Ampere’s law is identical to Weinberg-Salam theory
  \[ (g^{1/4} f^{\omega\tau})_{,\omega} - \sqrt{-2\Lambda_b} g^{1/4} [f^{\omega\tau}, A_\omega] = 4\pi g^{1/4} j^\tau \]  
  \[ (6) \]

- This determines the value of $\Lambda_b$ and $\Lambda_z$,
  \[ -\Lambda_z \approx \Lambda_b = \frac{\alpha}{8 l_P^2 \sin^2 \theta_w} \sim 1/l_P^2, \]  
  \[ (7) \]
  and this is consistent with $\Lambda_z$ being caused by zero-point fluctuations

- Other field equations have tiny extra terms
  \[ f_{\nu\mu} = 2A_{[\mu,\nu]} + \sqrt{-2\Lambda_b} [A_\nu, A_\mu] + (f^2)\Lambda_b^{-1/2} + (f'')\Lambda_b^{-1} \ldots \]  
  \[ (8) \]

  \[ G_{\nu\mu} = 8\pi T_{\nu\mu} + tr \left( f^\sigma (\nu f_{\mu})_\sigma - \frac{1}{4} g_{\nu\mu} f^{\rho\sigma} f_{\sigma\rho} \right) + \Lambda g_{\nu\mu} + (f^3)\Lambda_b^{-1/2} + (f'f')\Lambda_b^{-1} \ldots \]  
  \[ (9) \]
Non-Abelian LRES theory \(\approx\) Einstein-Weinberg-Salam theory

- Extra terms in the field equations are \(<10^{-13}\) of usual terms for worst-case \(|f_{\mu\nu}|\), \(|f_{\mu\nu;\alpha}|\) and \(|f_{\mu\nu;\alpha;\beta}|\) accessible to measurement.


- Usual Lorentz-force equation results from divergence of Einstein equations

- Exact solutions:
  - EM plane-wave solution is identical to that of Einstein-Maxwell theory.
  - Charged solution and Reissner-Nordström sol. have tiny fractional difference: \(10^{-76}\) for \(r=Q=M=M_\odot\), \(10^{-64}\) for \(r=10^{-17}\text{cm}, Q=e, M=M_e\).

- Standard tests

<table>
<thead>
<tr>
<th>test case</th>
<th>fractional difference from Einstein-Maxwell result</th>
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<tbody>
<tr>
<td>periastron advance</td>
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</tr>
<tr>
<td>deflection of light</td>
<td>(10^{-79})</td>
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<tr>
<td>time delay of light</td>
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<td></td>
<td>(Q=M=M_\odot, r=4M)</td>
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<td>(Q=e, M=M_P, r=a_0)</td>
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<td>(10^{-57})</td>
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<td>(10^{-56})</td>
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- Possible Proca field ghost with \(M_{Proca}/\hbar=\sqrt{2\Lambda_b}\sim 1/l_P\), but probably not
Lagrangian has $U(1) \otimes SU(2)$ invariance like Weinberg-Salam theory

- $\mathcal{L} \rightarrow \mathcal{L}$ under $SU(2)$ gauge transformation, with $2 \times 2$ matrix $U$
  \[ A_\nu \rightarrow U A_\nu U^{-1} + \frac{i}{\sqrt{2} \Lambda_b} U_{\nu\mu} U^{-1}, \]  
  \[ \Rightarrow \hat{\Gamma}^\alpha_{\nu\mu} \rightarrow U \hat{\Gamma}^\alpha_{\nu\mu} U^{-1} + 2 \delta^\alpha_{[\nu} U_{,\mu]} U^{-1}, \]  
  \[ \Rightarrow \hat{R}_{\nu\mu} \rightarrow U \hat{R}_{\nu\mu} U^{-1} \]  
  \[ N_{\nu\mu} \rightarrow U N_{\nu\mu} U^{-1}, \quad g_{\nu\mu} \rightarrow U g_{\nu\mu} U^{-1}, \quad f_{\nu\mu} \rightarrow U f_{\nu\mu} U^{-1}. \]  

- $\mathcal{L} \rightarrow \mathcal{L}$ under $U(1)$ gauge transformation, with scalar $\varphi$
  \[ A_\nu \rightarrow A_\nu + \frac{1}{\sqrt{2} \Lambda_b} \varphi, \]  
  \[ \Rightarrow \hat{\Gamma}^\alpha_{\nu\mu} \rightarrow \hat{\Gamma}^\alpha_{\nu\mu} - 2 i I \delta^\alpha_{[\nu} \varphi_{,\mu]}, \]  
  \[ \Rightarrow \hat{R}_{\nu\mu} \rightarrow \hat{R}_{\nu\mu} \]  
  \[ N_{\nu\mu} \rightarrow N_{\nu\mu}, \quad g_{\nu\mu} \rightarrow g_{\nu\mu}, \quad f_{\nu\mu} \rightarrow f_{\nu\mu}. \]  

- $\mathcal{L}^\ast = \mathcal{L}$ when $A_\nu$ and $f_{\nu\mu}$ are Hermitian
  \[ \hat{\Gamma}^\alpha_{\nu\mu} = \hat{\Gamma}^\alpha_{\mu\nu}^T, \quad \hat{R}_{\nu\mu} = \hat{R}_{\mu\nu}^T, \quad N_{\nu\mu}^* = N_{\mu\nu}^T, \quad N^* = N, \]  
  \[ \Rightarrow A^\ast_\nu = A_\nu^T, \quad f^*_{\nu\mu} = f_{\nu\mu}^T, \quad g^*_{\nu\mu} = g_{\nu\mu}^T, \quad g^* = g. \]
Summary of non-Abelian $\Lambda$-renormalized Einstein-Schrödinger theory

- \( (\text{non–Abelian LRES theory}) \approx (\text{Einstein–Weinberg–Salam theory}) \) with \( |\Lambda_z| \approx \frac{\alpha}{8 l_P^2 \sin^2 \theta_w} \sim \frac{1}{l_P^2} \).

- Extra terms in Weinberg-Salam field equations are \(< 10^{-13}\) of usual terms.

- \( \lim_{|\Lambda_z| \to \infty} (\text{LRES theory}) = (\text{Einstein–Maxwell theory}) \) (Gen.Rel.Grav.(Online First) gr-qc/0801.2307)

- Lagrangian has \( U(1) \otimes SU(2) \) invariance like Weinberg-Salam theory.

- \( \mathcal{L}_m \) contains \( \psi_{\nu}, \phi \) fields, and could also include rest of Standard Model.

- It’s the ES theory but with Hermitian matrix components and a \( \Lambda_z \) term.

- Suggests untried approaches to a complete unified field theory:
  - Larger matrices: \( 5 \times 5 \) matrices for \( SU(5) \) or \( U(1) \otimes SU(5) \)?

- For details see gr-qc/0804.1962
Backup charts
The non-Abelian Ricci tensor

- We use one of many nonsymmetric generalizations of Ricci tensor

\[ \hat{R}_{\nu\mu} = \hat{\Gamma}_{\nu,\alpha} - \hat{\Gamma}_{(\alpha(\nu),\mu)} + \frac{1}{2} \hat{\Gamma}_{\nu\mu} \hat{\Gamma}_{(\sigma\alpha)} + \frac{1}{2} \hat{\Gamma}_{(\sigma\alpha)} \hat{\Gamma}_{\nu\mu} - \hat{\Gamma}_{\nu\alpha} \hat{\Gamma}_{\sigma\mu} - \hat{\Gamma}_{[\tau\nu]} \hat{\Gamma}_{[\rho\mu]} / 3 \]  
(20)

- Because it has important invariance properties

\[ R_{\rho\tau}(\hat{\Gamma}_{\nu\mu}^T) = R_{\tau\rho}^T(\hat{\Gamma}_{\mu\nu}) \quad \text{(transposition symmetric)} \]  
(21)

\[ R_{\rho\tau}(U\hat{\Gamma}_{\nu\mu} U^{-1} + 2\delta_{[\nu}^\alpha U_{,\mu]} U^{-1}) = UR_{\rho\tau}(\hat{\Gamma}_{\nu\mu}) U^{-1} \quad \text{(almost SU(2) invariant)} \]  
(22)

\[ R_{\rho\tau}(\hat{\Gamma}_{\nu\mu} - 2iI \delta_{[\nu}^\alpha \delta_{,\mu]}^\varphi) = R_{\rho\tau}(\hat{\Gamma}_{\nu\mu}) \quad \text{(U(1) invariant)} \]  
(23)

- For Abelian fields the third and fourth terms are the same.

- Reduces to the ordinary Ricci tensor for \( \hat{\Gamma}_{[\nu\mu]} = 0 \) and \( \hat{\Gamma}_{\alpha[\nu,\mu]} = 0 \), as occurs in ordinary general relativity.
The Lagrangian Density Again

• $A_\nu$ is defined by
  \[ A_\nu = \frac{\hat{\rho}_\nu}{\sqrt{-18\Lambda_b}}. \]  \hspace{1cm} (24)

• $\hat{\Gamma}_\nu^\mu$ can be decomposed into $\bar{\Gamma}_\nu^\mu$ with the symmetry $\bar{\Gamma}_\nu^\alpha = \bar{\Gamma}_\alpha^\nu$, and $A_\nu$,
  \[ \bar{\Gamma}_\nu^\mu = \hat{\Gamma}_\nu^\mu + \frac{1}{3} (\delta_\mu^\alpha \hat{\Gamma}_{\sigma [\nu} - \delta_\nu^\alpha \hat{\Gamma}_{\sigma [\mu}) \Rightarrow \hat{\Gamma}_\nu^\mu = \bar{\Gamma}_\nu^\mu + 2\delta_\nu^\mu A_\nu \sqrt{-2\Lambda_b}. \]  \hspace{1cm} (25)

• The Lagrangian density (3) in terms of $A_\mu$, $\bar{\Gamma}_\nu^\mu$, and $\bar{\Gamma}_\nu^\mu = \hat{\mathcal{R}}_{\nu\mu}(\bar{\Gamma})$ is,
  \[ \mathcal{L}(\hat{\mathcal{R}}_{\rho\tau}, N_{\rho\tau}) = -\frac{1}{16\pi} N^{1/4} [tr(N^{-1}\nu\nu(\bar{\mathcal{R}}_{\nu\mu} + 2A_{[\nu,\mu]}\sqrt{-2\Lambda_b}) + 2\Lambda_b[A_\nu, A_\mu]) + 4\Lambda_b] \]
  \[ -\frac{1}{4\pi} \Lambda g^{1/4} + \mathcal{L}_m(A_\nu, g_{\mu\nu}, \psi_{e\nu}, \phi, \cdots). \]  \hspace{1cm} (26)

• The nonsymmetric Ricci tensor (20) reduces to
  \[ \bar{\mathcal{R}}_{\nu\mu} = \bar{\Gamma}_{\nu\mu,\alpha} - \bar{\Gamma}_{\alpha(\nu,\mu)} + \frac{1}{2} \bar{\Gamma}_{\nu\mu} \bar{\Gamma}_{\sigma\alpha} + \frac{1}{2} \bar{\Gamma}_{\sigma\alpha} \bar{\Gamma}_{\nu\mu} - \bar{\Gamma}_{\nu\alpha} \bar{\Gamma}_{\sigma\mu} \]  \hspace{1cm} (27)

• We assume the special case $g_{\nu\mu} = \text{Itr}(g_{\nu\mu})/2$ and $\bar{\Gamma}_{\nu\mu} = \text{Itr}(\bar{\Gamma}_{\nu\mu})/2$. 
The Einstein Equations

- \( g_{\mu\nu} \) and \( f_{\mu\nu} \) are defined by (with \( c=G=1 \))
  \[
  g^{1/4} g^{\nu\mu} = N^{1/4} N^{-1(\mu\nu)}
  \] (28)
  \[
  g^{1/4} f^{\nu\mu} = i N^{1/4} N^{-1[\nu\mu]} \Lambda_b^{1/2} / \sqrt{2}
  \] (29)

Inverting these definitions gives (after some effort)

\[
N_{(\nu\mu)} = g_{\nu\mu} - 2 \left( f^\sigma (u f_\sigma) - \frac{1}{4} g_{\nu\mu} tr(f^\rho f^\sigma) / 2 \right) \Lambda_b^{-1} + (f^3) \Lambda_b^{-3/2} \cdots \] (30)
\[
N_{[\nu\mu]} = f_{\nu\mu} \sqrt{2} i \Lambda_b^{-1/2} + (f^2) \Lambda_b^{-1} \cdots \] (31)

- \( f_{\mu\nu} \) comes from \( \delta \mathcal{L} / \delta (\sqrt{-NN^{-1(\mu\nu)}}) = 0 \) and \( \tilde{\mathcal{R}}_{[\nu\mu]} = (f'') \Lambda_b^{-1/2} \cdots \) from (43),

\[
N_{[\nu\mu]} = 2 A_{[\mu,\nu]} \sqrt{-2} \Lambda_b^{-1/2} - 2 [A_\nu, A_\mu] - \tilde{\mathcal{R}}_{[\nu\mu]} \Lambda_b^{-1}
\]
\[
\Rightarrow \quad f_{\nu\mu} = A_{\mu,\nu} - A_{\nu,\mu} + \sqrt{-2} \Lambda_b [A_\nu, A_\mu] + (f^2) \Lambda_b^{-1/2} + (f'') \Lambda_b^{-1} \cdots \] (33)

- Einstein equations come from \( \delta \mathcal{L} / \delta (\sqrt{-NN^{-1(\mu\nu)}}) = 0 \),

\[
\tilde{G}_{\nu\mu} = 8\pi T_{\nu\mu} - \Lambda_b tr \left( N_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} N^\rho_\rho \right) + \Lambda g_{\nu\mu}
\]
\[
= 8\pi T_{\nu\mu} + tr \left( f^\sigma (u f_\sigma) - \frac{1}{4} g_{\nu\mu} f^\rho f^\sigma f_\rho \right) + \Lambda g_{\nu\mu} + (f^3) \Lambda_b^{-1/2} + (f'f') \Lambda_b^{-1} \cdots \] (35)
Weinberg-Salam equivalent of Ampere’s Law

- Maxwell’s equations come from $\delta \mathcal{L}/\delta A_\tau = 0$,

$$\left(g^{1/4} f^{\omega \tau}\right)_\omega - \sqrt{-2\Lambda_b} g^{1/4} [f^{\omega \tau}, A_\omega] = 4\pi g^{1/4} j^\tau,$$

where $f_{\nu \mu} \approx 2 A_{[\mu, \nu]} + \sqrt{-2\Lambda_b} [A_\nu, A_\mu]$ and

$$j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta A_\tau}.$$  \hfill (37)

- $\mathcal{L}_m$ contains $\psi_{e\nu}, \phi$ fields of Weinberg-Salam theory,

$$j^\tau = Q \bar{\psi}_{e\nu} \gamma^\tau \psi_{e\nu}.$$  \hfill (38)
The Connection Equations

- Relation between $\tilde{\alpha}_{\mu\nu}$ and $N_{\mu\nu}$ like $(\sqrt{-g}g^{\tau\rho})_{,\beta}=0$ comes from $\delta L/\delta \tilde{\alpha}^\beta_{\tau\rho}=0$,

$$tr[(N^{1/4}N^{-1}\rho_{\tau})_{,\beta} + \tilde{\alpha}_{\sigma\beta}N^{1/4}N^{-1}\rho_{\sigma} + \tilde{\alpha}_{\beta\sigma}N^{1/4}N^{-1}\sigma_{\tau} - \tilde{\alpha}_{\beta\alpha}N^{1/4}N^{-1}\rho_{\tau}] = \frac{8\pi\sqrt{2}i}{3}g^{1/4} tr[j^{[\rho}\delta^{\tau]}_\beta] \Lambda_b^{-1/2}. \quad (39)$$

- Solving these equations gives

$$\tilde{\alpha}_{(\nu\mu)} = \frac{I}{2} g^{\alpha\rho}(g_{\mu\rho,\nu} + g_{\rho\nu,\mu} - g_{\nu\mu,\rho}) + (f'f)\Lambda_b^{-1} \ldots, \quad (40)$$

$$\tilde{\alpha}_{[\nu\mu]} = (f')\Lambda_b^{-1} \ldots, \quad (41)$$

$$\tilde{R}_{(\nu\mu)} = R_{\nu\mu} + (f'f')\Lambda_b^{-1} \ldots, \quad (42)$$

$$\tilde{R}_{[\nu\mu]} = (f'')\Lambda_b^{-1/2} \ldots. \quad (43)$$

$\Rightarrow \tilde{R}_{(\nu\mu)} \approx R_{\nu\mu}$ and $f_{\nu\mu} \approx 2A_{[\mu,\nu]} + \sqrt{-2\Lambda_b}[A_{\nu},A_{\mu}]$ with fractional differences $<10^{-13}$ for worst-case $|f_{\mu\nu}|, |f_{\mu\nu;\alpha}|, |f_{\mu\nu;\alpha;\beta}|$ accessible to measurement (e.g. $10^{20}eV, 10^{34}Hz\gamma$-rays).
Proca waves as Pauli-Villars ghosts?

• If wave-packet Proca waves exist and if they have negative energy, perhaps the Proca field functions as a built-in Pauli-Villars ghost

\[ \omega_c = \omega_{\text{Proca}} = \sqrt{2\Lambda_b}, \quad -\Lambda_z \approx \Lambda_b = \frac{\alpha}{8l_P^2\sin^2\theta_w} \]  

\[ \Lambda_z = -\frac{\omega_c^4l_P^2}{2\pi} \left( \begin{array}{c} \text{fermion}\ \text{spin states} \\ \text{boson}\ \text{spin states} \end{array} \right) \]  

\[ \Rightarrow \left( \begin{array}{c} \text{fermion}\ \text{spin states} \\ \text{boson}\ \text{spin states} \end{array} \right) = \frac{4\pi\sin^2\theta_w}{\alpha} = 412.8 \pm 2 \]  

• In this case $4\pi\sin^2\theta_w/\alpha$ or its “bare” value at $\omega_c$ should be an integer.

• For the Standard Model the difference in (46) is about 60.

• Non-Abelian LRES theory works for $d \times d$ instead of $2 \times 2$ matrices.

• Perhaps some value of “d” is consistent with (46).

• $SU(5)$ almost unifies the Standard Model, how about $U(1) \otimes SU(5)$?