Λ-renormalized Einstein-Schrödinger theory: an alternative to Einstein-Maxwell theory

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L-renormalized Einstein-Schrödinger (LRES) theory

- Einstein-Maxwell theory can be derived from a Palatini Lagrangian density,

\[
\mathcal{L}(\Gamma^\lambda_{\rho\tau}, g_{\rho\tau}, A_\nu) = -\frac{1}{16\pi} \left[ \sqrt{-g} g^{\mu\nu} R_{\nu\mu} (\Gamma) + 2\Lambda \sqrt{-g} \right] + \frac{1}{4\pi} \sqrt{-g} A_{[\alpha,\beta]} g^{\alpha\mu} g^{\beta\nu} A_{[\mu,\nu]} + \mathcal{L}_m (u^\nu, \psi_e, A_\nu, g_{\mu\nu}, \cdots). \tag{1}
\]

- LRES theory uses nonsymmetric \( \hat{\Gamma}^\alpha_{\mu\nu} \) and \( N_{\mu\nu} \), excludes \( \sqrt{-g} A_{[\alpha,\beta]} g^{\alpha\mu} g^{\beta\nu} A_{[\mu,\nu]} \), and includes \( \Lambda_z \) from zero-point fluctuations,

\[
\mathcal{L}(\hat{\Gamma}^\lambda_{\rho\tau}, N_{\rho\tau}) = -\frac{1}{16\pi} \left[ \sqrt{-N} N^{-1\mu\nu} R_{\nu\mu} (\hat{\Gamma}) + 2\Lambda_b \sqrt{-N} \right] - \frac{1}{16\pi} 2\Lambda_z \sqrt{-g} + \mathcal{L}_m (u^\nu, \psi_e, A_\nu, g_{\mu\nu}, \cdots), \quad N = \det(N_{\mu\nu}) \tag{2}
\]

where the “bare” \( \Lambda_b \approx -\Lambda_z \) so the “physical” \( \Lambda = \Lambda_b + \Lambda_z \) matches measurement, and the metric \( g_{\mu\nu} \) and potential \( A_\nu \) are defined by

\[
\sqrt{-g} g^{\nu\mu} = \sqrt{-N} N^{-1(\mu\nu)}, \quad A_\nu = \hat{\Gamma}^\rho_{[\nu\rho]} / \sqrt{-18\Lambda_b}, \quad (\text{with } c = G = 1). \tag{3}
\]

- \( \lim_{|\Lambda_z| \to \infty} \left( \text{LRES theory} \right) = \left( \text{Einstein–Maxwell theory} \right) \) but \( \omega_c \sim \frac{1}{l_P} \Rightarrow |\Lambda_z| \sim \omega_c^4 l_P^2 \sim \frac{1}{l_P^2} \).
LRES theory avoids the problems of Einstein-Schrödinger theory

- Matches measurement as well as Einstein-Maxwell theory.

- Definitely predicts a Lorentz force:
  - Usual Lorentz force equation results from divergence of Einstein equations,
  - Lorentz force also results from the EIH method, with $L_m = 0$.

- Avoids ghosts:
  - With a cutoff frequency $\omega_c \sim 1/l_P$ we have $\Lambda_z \sim -\omega_c^4 l_P^2$ (with $c=G=1$),
  - Ghosts are cut off because they would have $\omega_{\text{ghost}} = \sqrt{-2\Lambda_z} \sim \sqrt{2}\omega_c^2 l_P > \omega_c$,
  - If we fully renormalize with $\omega_c \to \infty$ then $\omega_{\text{ghost}} \to \infty$, meaning no ghost.

- Well motivated:
  - It’s a vacuum energy renormalization of Einstein-Schrödinger theory,
  - $\Lambda_z\sqrt{-g}$ term should be expected to occur as a quantization effect,
  - Zero-point fluctuations are essential to QED - they cause the Casimir effect,
  - $\Lambda = \Lambda_b + \Lambda_z$ is similar to mass/charge/field-strength renormalization in QED,
  - $\Lambda_z\sqrt{-g}$ modification has never been considered before.
LRES theory matches measurement as well as Einstein-Maxwell theory

- Reduces to ordinary GR without electromagnetism for symmetric fields.

- Extra terms in Einstein and Maxwell equations are \( <10^{-16} \) of usual terms for worst-case \( |F_{\mu\nu}|, |F_{\mu\nu;\alpha}| \) and \( |F_{\mu\nu;\alpha;\beta}| \) accessible to measurement.

- Exact solutions:
  - EM plane-wave solution is identical to that of Einstein-Maxwell theory.
  - Charged solution and Reissner-Nordström sol. have tiny fractional difference: \( 10^{-76} \) for \( r=Q=M=M_\odot \), \( 10^{-64} \) for \( r=10^{-17}cm, Q=e, M=M_e \).

- Standard tests: fractional difference from Einstein-Maxwell result

<table>
<thead>
<tr>
<th>test case →</th>
<th>extremal charged black hole</th>
<th>atomic parameters</th>
</tr>
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<tr>
<td>periastron advance</td>
<td>( Q=M=M_\odot, r=4M )</td>
<td>( Q=e, M=M_P, r=a_0 )</td>
</tr>
<tr>
<td>deflection of light</td>
<td>( 10^{-78} )</td>
<td>( 10^{-91} )</td>
</tr>
<tr>
<td>time delay of light</td>
<td>( 10^{-79} )</td>
<td>( 10^{-57} )</td>
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- Other Standard Model fields can be added just like Einstein-Maxwell theory:
  - Energy levels of Hydrogen atom have fractional difference of \( <10^{-90} \).
Why pursue LRES theory if it’s so close to Einstein-Maxwell theory?

- It unifies gravitation and electromagnetism in a classical sense.
- Quantization of LRES theory is untried approach to quantization of gravity:
  - LRES theory gets much different than Einstein-Maxwell theory as $k \to 1/l_P$,
  - This could possibly fix some infinities which spoil the quantization of GR.
- LRES theory suggests untried approaches to a complete unified field theory:
  - Higher dimensions, but with LRES theory instead of vacuum GR?
  - Non-abelian fields, but with LRES theory instead of Einstein-Maxwell?
- We still don’t have a unified field theory, 50 years after Einstein:
  - Standard Model: excludes gravity, 25 parameters, not very “beautiful”
  - String theory: background dependent, spin-2 particle $\Rightarrow$ GR?, $10^{500}$ versions,
    problems accounting for $\Lambda > 0$ and broken symmetry, little predictive ability.
Summary of $\Lambda$-renormalized Einstein-Schrödinger theory

- $\lim_{|\Lambda_z| \to \infty} (\text{LRES theory}) = (\text{Einstein–Maxwell theory})$ but $\omega_c \sim \frac{1}{l_P} \Rightarrow |\Lambda_z| \sim \omega_c^4 l_P^2 \sim \frac{1}{l_P^2}$.
- Matches measurement as well as Einstein-Maxwell theory.
- Reduces to ordinary GR without electromagnetism for symmetric fields.
- Other Standard Model fields can be added just like Einstein-Maxwell theory.
- Avoids the problems of the original Einstein-Schrödinger theory.
- Well motivated - it’s the ES theory but with a quantization effect.
- Unifies gravitation and electromagnetism in a classical sense.
- Suggests untried approaches to a complete quantized unified field theory.
Backup charts
The Lagrangian Density Again

• $A_\nu$ and $F_{\mu\nu}$ are defined by (with $c=G=1$)

$$A_\nu = \hat{\Gamma}^\rho_{[\nu\rho]} / \sqrt{-18\Lambda_b}, \quad F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}. \quad (4)$$

• $\hat{\Gamma}^\alpha_{\nu\mu}$ can be decomposed into $\tilde{\Gamma}^\alpha_{\nu\mu}$ with the symmetry $\tilde{\Gamma}^\alpha_{\nu\alpha} = \tilde{\Gamma}^\alpha_{\alpha\nu}$, and $A_\nu$,

$$\tilde{\Gamma}^\alpha_{\nu\mu} = \hat{\Gamma}^\alpha_{\nu\mu} + (\delta^\alpha_\mu \hat{\Gamma}^\sigma_{[\sigma\nu]} - \delta^\alpha_\nu \hat{\Gamma}^\sigma_{[\sigma\mu]}) / 3 \Rightarrow \hat{\Gamma}^\alpha_{\nu\mu} = \tilde{\Gamma}^\alpha_{\nu\mu} + 2\delta^\alpha_{[\mu} A_{\nu]} \sqrt{-2\Lambda_b^{1/2}}. \quad (5)$$

• The “Hermitianized Ricci tensor” in (2) reduces to the ordinary Ricci tensor for symmetric fields with $\Gamma^\alpha_{[\nu\mu]} = 0$ and $\Gamma^\alpha_{\alpha[\nu,\mu]} = R^\alpha_{\alpha\mu\nu}/2 = 0$,

$$R_{\nu\mu}(\hat{\Gamma}) = \hat{\Gamma}^\alpha_{\nu\mu,\alpha} - \hat{\Gamma}^\alpha_{(\alpha(\nu),\mu)} + \hat{\Gamma}^\rho_{\nu\mu} \hat{\Gamma}^\alpha_{(\rho\alpha)} - \hat{\Gamma}^\rho_{\nu\alpha} \hat{\Gamma}^\rho_{\mu} - \hat{\Gamma}^\tau_{[\tau\nu]} \hat{\Gamma}^\alpha_{[\alpha\mu]}/3. \quad (6)$$

• $R_{\nu\mu}$ exhibits both charge conjugation symmetry and gauge invariance

$$R_{\mu\nu}(\hat{\Gamma}^T) = R_{\nu\mu}(\hat{\Gamma}), \quad R_{\nu\mu}(\hat{\Gamma}^\alpha_{\rho\tau} + \delta^\alpha_{[\rho} \phi_{,\tau]}) = R_{\nu\mu}(\tilde{\Gamma}^\alpha_{\rho\tau}). \quad (7)$$

• The Lagrangian density (2) in terms of $A_\mu$, $\tilde{\Gamma}^\alpha_{\nu\mu}$ and $\bar{R}_{\nu\mu} = R_{\nu\mu}(\tilde{\Gamma})$ is,

$$\mathcal{L}(\hat{\Gamma}^\lambda_{\rho\tau}, N_{\rho\tau}) = -\frac{1}{16\pi} \left[ \sqrt{-N} N^{-1\mu\nu} (\bar{R}_{\nu\mu} + 2A_{[\nu,\mu]} \sqrt{-2\Lambda_b^{1/2}}) + 2\Lambda_b \sqrt{-N} \right]$$

$$-\frac{1}{16\pi} 2\Lambda_g \sqrt{-g} + \mathcal{L}_m(u^\nu, \psi_e, A_\nu, g_{\mu\nu}, \cdots). \quad (8)$$
The Einstein Equations

- $g_{\mu\nu}$ and $f_{\mu\nu}$ are defined by (with $c=G=1$)
  \[ \sqrt{-g} g^{\nu\mu} = \sqrt{-NN^{-1}(\mu\nu)}, \]
  \[ \sqrt{-g} f^{\nu\mu} = \sqrt{-NN^{-1}[\mu\nu]} \Lambda_b^{1/2}/\sqrt{-2}. \]  
  Inverting these definitions gives (after some effort)
  \[ N_{(\nu\mu)} = g_{\nu\mu} - 2 \left( f_{\nu}^{\alpha} f_{\alpha\mu} - \frac{1}{4} g_{\nu\mu} f_{\rho\alpha} f_{\alpha\rho} \right) \Lambda_b^{-1} + O(\Lambda_b^{-2}), \]
  \[ N_{[\nu\mu]} = f_{\nu\mu} \sqrt{-2} \Lambda_b^{-1/2} + O(\Lambda_b^{-3/2}). \]
  - $f_{\mu\nu} \approx F_{\mu\nu}$ comes from $\delta \mathcal{L}/\delta(\sqrt{-NN^{-1}[\mu\nu]}))=0$ and $\tilde{R}_{[\nu\mu]}=O(\Lambda_b^{-1/2})$ from (26),
    \[ N_{[\nu\mu]} = 2A_{[\mu,\nu]} \sqrt{-2} \Lambda_b^{-1/2} - \tilde{R}_{[\nu\mu]} \Lambda_b^{-1}, \]
    \[ \Rightarrow f_{\nu\mu} = A_{\mu,\nu} - A_{\nu,\mu} + O(\Lambda_b^{-1}). \]
  - Einstein equations come from $\delta \mathcal{L}/\delta(\sqrt{-NN^{-1}(\mu\nu)})=0$,
    \[ \tilde{R}_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} \tilde{R}_{\rho}^{\rho} = 8\pi T_{\nu\mu} - \Lambda_b \left( N_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} N_{\rho}^{\rho} \right) + \Lambda_z g_{\nu\mu} \]
    \[ = 8\pi T_{\nu\mu} + 2 \left( f_{\nu}^{\alpha} f_{\alpha\mu} - \frac{1}{4} g_{\nu\mu} f_{\rho\alpha} f_{\alpha\rho} \right) + \Lambda g_{\nu\mu} + O(\Lambda_b^{-1}). \]
Maxwell’s Equations

- Maxwell’s equations come from $\delta \mathcal{L}/\delta A_\tau = 0$ and $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$,

\[
  f_{\nu\tau}^{\mu} = 4\pi j^\tau, \\
  F_{[\mu\nu,\alpha]} = 0,
\]  

(17)  

(18)

where $f_{\mu\nu} \approx F_{\mu\nu}$ and

\[
  j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta \mathcal{L}_m}{\delta A_\tau}.
\]  

(19)

- $\mathcal{L}_m$ may contain other fields just like Einstein-Maxwell theory,

\[
  j^\tau = Q \bar{\psi} \gamma^\tau \psi_e \quad \text{for spin–1/2}, \\
  j^\tau = \rho u^\tau \quad \text{for classical hydrodynamics}.
\]  

(20)  

(21)
The Connection Equations

- Relation between $\tilde{\Gamma}^\alpha_{\mu\nu}$ and $N_{\mu\nu}$ like $\left(\sqrt{-g}g^{\tau\rho}\right)_{,\beta} = 0$ comes from $\delta\mathcal{L}/\delta\tilde{\Gamma}_{\tau\rho}^\beta = 0$,

\[
(\sqrt{-N}N^{-1}\rho\tau)_{,\beta} + \tilde{\Gamma}_{\nu\beta}^\tau \sqrt{-N}N^{-1}\rho\nu + \tilde{\Gamma}_{\beta\nu}^\rho \sqrt{-N}N^{-1}\nu\tau - \tilde{\Gamma}_{\beta\alpha}^\rho \sqrt{-N}N^{-1}\rho\tau = \frac{8\pi}{3} \sqrt{-g} j_{[\rho} \delta_{\beta]}^{\tau} \sqrt{-2} \Lambda_b^{-1/2}. \quad (22)
\]

- Solving these equations gives

\[
\tilde{\Gamma}^\alpha_{(\nu\mu)} = \frac{1}{2} g^{\alpha\rho} (g_{\mu\rho,\nu} + g_{\rho\nu,\mu} - g_{\nu\mu,\rho}) + O(\Lambda_b^{-1}), \quad (23)
\]

\[
\tilde{\Gamma}^\alpha_{[\nu\mu]} = O(\Lambda_b^{-1/2}), \quad (24)
\]

\[
\tilde{\mathcal{R}}_{(\nu\mu)} = R_{\nu\mu} + \text{(terms like } f^{\alpha\tau} f_{\tau(\mu;\nu);\alpha} \Lambda_b^{-1} \text{ and } f^\rho_{\mu;\alpha} f^{\alpha\nu;\rho} \Lambda_b^{-1}) \text{)}, \quad (25)
\]

\[
\tilde{\mathcal{R}}_{[\nu\mu]} = \text{(terms like } f_{[\mu\nu,\tau]}^{\tau} \Lambda_b^{-1/2}, f^{\tau}_{[\mu;\nu];\tau} \Lambda_b^{-1/2} \text{ and } j_{[\nu,\mu]} \Lambda_b^{-1/2}). \quad (26)
\]

⇒ $\tilde{\mathcal{R}}_{(\nu\mu)} \approx R_{\nu\mu}$ and $f_{\nu\mu} \approx F_{\nu\mu}$ with fractional differences $<10^{-16}$ for worst-case $|f_{\mu\nu}|, |f_{\mu\nu;\alpha}|, |f_{\mu\nu;\alpha;\beta}|$ accessible to measurement (e.g. $10^{20} eV, 10^{34} Hz \gamma$-rays).
The Generalized Contracted Bianchi Identity

• A generalized contracted Bianchi identity results from (22),
  \[
  \left(\sqrt{-NN^{-1}\sigma\nu}\tilde{R}_{\nu\lambda} + \sqrt{-NN^{-1}\nu\sigma}\tilde{R}_{\lambda\nu}\right)_{,\sigma} - \sqrt{-NN^{-1}\sigma\nu}\tilde{R}_{\nu\sigma,\lambda} = 0. \tag{27}
  \]

• It may also be written in the manifestly covariant form,
  \[
  \left(\sqrt{-NN^{-1}\sigma\nu}\tilde{R}_{\nu\lambda} + \sqrt{-NN^{-1}\nu\sigma}\tilde{R}_{\lambda\nu}\right)_{,\sigma} - \sqrt{-NN^{-1}\sigma\nu}\tilde{R}_{\nu\sigma,\lambda} = 0, \tag{28}
  \]

• Or in a third form,
  \[
  \tilde{G}^{\sigma}_{\lambda;\sigma} = \left(\frac{3}{2} f^{\sigma\nu\rho} \tilde{R}_{[\sigma\nu,\lambda]} + 4\pi j^{\nu} \tilde{R}_{[\nu\lambda]}\right) \sqrt{-2} \Lambda_b^{-1/2}, \tag{29}
  \]
  where
  \[
  \tilde{G}_{\nu\mu} = \tilde{R}_{(\nu\mu)} - \frac{1}{2} g_{\nu\mu} \tilde{R}_{\rho}^{\rho}. \tag{30}
  \]

• The usual contracted Bianchi identity \(2(\sqrt{-g}g^{\sigma\nu}R_{\nu\lambda}),_{\sigma} - \sqrt{-g}g^{\sigma\nu}R_{\nu\sigma,\lambda} = 0,\) or \(G^{\sigma}_{\lambda;\sigma} = 0\) is also valid.
The Lorentz Force Equation

- Lorentz force equation comes from divergence of the Einstein equations (15)
  \[ T_{\mu;\nu} = F_{\mu\nu} j^\nu \]  \hspace{1cm} (31)

  where

  \[ j^\tau = \frac{-1}{\sqrt{-g}} \frac{\delta L_m}{\delta A^\tau}, \]  \hspace{1cm} (32)

  \[ T_{\mu\nu} = S_{\mu\nu} - \frac{1}{2} g_{\mu\nu} S^\alpha \]  \hspace{1cm} (33)

  \[ S_{\mu\nu} = \frac{2 \delta L_m}{\delta (\sqrt{-gg^{\mu\nu}})}. \]  \hspace{1cm} (34)

- Here we have used equations (29,17,13) and the following identity which can be derived using only the definitions of $g_{\mu\nu}$ and $f_{\mu\nu}$,

  \[ \left( N^{(\mu}_{\sigma)} - \frac{1}{2} \delta^{\mu}_{\sigma} N^\rho_{\rho} \right)_{;\mu} = \left( \frac{3}{2} f^{\nu\rho} N_{[\nu\rho,\sigma]} + f^{\nu\rho} ;_{\nu} N_{[\rho\sigma]} \right) \sqrt{-2} \Lambda_b^{-1/2}. \]  \hspace{1cm} (35)

- Covariant derivative "\; ;\;" is always done using the Christoffel connection formed from the symmetric metric $g_{\mu\nu}$. 
An Exact Charged Solution

- This charged solution is very close to the Reissner-Nordström solution,

\[ g_{\nu\mu} = \tilde{c} \begin{pmatrix} a & -1/ac^2 \\ -r^2 & -r^2 \sin^2 \theta \end{pmatrix}, \]

(36)

\[ f_{\nu\mu} = \frac{1}{\tilde{c}} \begin{pmatrix} 0 & Q/r^2 \\ -Q/r^2 & 0 \end{pmatrix}, \]

(37)

\[ A_0 = \frac{Q}{r} \left[ 1 + \frac{M}{\Lambda_b r^3} - \frac{4Q^2}{5\Lambda_b r^4} + \mathcal{O}(\Lambda_b^{-2}) \right], \]

(38)

where

\[ a = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \left[ 1 + \frac{Q^2}{10\Lambda_b r^4} + \mathcal{O}(\Lambda_b^{-2}) \right], \quad \tilde{c} = \sqrt{1 - \frac{2Q^2}{\Lambda_b r^4}}. \]

(39)

- Additional terms are tiny for worst-case radii accessible to measurement:
  - \( Q^2/\Lambda_b r^4 \sim 10^{-76} @ r = Q = M = M_\odot; \sim 10^{-64} @ r = 10^{-17} \text{cm}, Q = e, M = M_e, \)
  - \( M/\Lambda_b r^3 \sim 10^{-76} @ r = Q = M = M_\odot; \sim 10^{-70} @ r = 10^{-17} \text{cm}, Q = e, M = M_e. \)